Math 4321

3/17/15

Here is the proof of the result in Section 27 that we did not prove in class today.

Let F = field, $R = F[x]$, $p \in R^{\times}$.

p is irred over F iff $\langle p \rangle$ = maximal ideal of R.

Proof

$\Leftarrow$ Suppose $\langle p \rangle$ is max. So $\langle p \rangle \neq R$ and $p \notin F$.

Let $p = fg$ for some $f, g \in R$. $\langle p \rangle$ max $\Rightarrow \langle p \rangle$ prime.

and $fg = p \in \langle p \rangle \Rightarrow f \in \langle p \rangle$ or $g \in \langle p \rangle$. $\Rightarrow p \mid f$ or

$p \mid g \Rightarrow \deg(p) \leq \deg(f)$ or $\deg(p) \leq \deg(g) \Rightarrow p$ irred.

$\Rightarrow$ Suppose p irred over F. Let H be an ideal with

$\langle p \rangle \subset H \subseteq R$. All ideals in R are principal $\Rightarrow \exists\ g \in H$ such

that $H = \langle g \rangle$. However, $p \in H = \langle g \rangle \Rightarrow p = gq$ for some $q \in R$.

p irred $\Rightarrow \deg(g) = 0$ or $\deg(q) = 0$. If $\deg(g) = 0$, then $g \in F^{\times}$

so $H = R$ $\not\Rightarrow$. If $\deg(q) = 0$, then $q \in F^{\times}$; so $g = pq^{-1} \in \langle p \rangle$.

Hence, $H = \langle g \rangle \subseteq \langle p \rangle$. $\not\Rightarrow$.