Test 1 will be in PKH 113 in class time, Tues Feb 17

- See above for date, time and location of Test 1. It will last 20 minutes and is worth 12% of your course grade.
- The material to be covered on Test 1 is the same as that covered on the homework through §2.2 inclusive. (Homework is listed at my website: www.uta.edu/math/vancliff/T/S15 .)
- For my office hours, see my website. In particular, there will be an office hour on the day of the test from 1:05 pm until 1:50 pm in my office, PKH 462.
- This test will be entirely multiple choice, but you do NOT need to bring a scantron form. There will be several choices of answer per multiple-choice question and, for each, only one answer will be the correct one. You should do rough work on the test or on paper provided by me. No calculator is allowed. No notes or cards are allowed.

BRING YOUR MYMAV ID CARD WITH YOU.

- When I write a test, I look over the lecture notes and homework which have already been assigned, and use them to model about 85% of the test problems (and most of them are fair game). You should expect 8-11 questions in total.
- A good way to review is to go over the homework problems you have not already done & make sure you understand all the homework well by 48 hours prior to the test. You could also go over the quiz; the quiz and its solutions are posted online at my website in a single pdf file (use password "math3330" to open the file); see http://www.uta.edu/math/vancliff/T/S15/3330qs.pdf . In addition, this information sheet provides some practice problems that are provided to help you study if you have finished all the homework questions. These practice questions do NOT form a model for the test. These questions (like those on the quiz) are intended only to help you identify any gaps in your understanding. In the last 24 hours before the test, reread ALL the homework problems, skim through the lecture notes, & go over these practice questions again. Consider adopting one or more of the study techniques linked at my website. Make sure you learn any formulae taught in class.
- It is your responsibility to be on time.
- Try to keep your eyes on your own work during the test.
- All nontest items will have to be put by the side walls or by my desk prior to the start of the test; this includes cell phones & tablets, which need to be switched off & in with your belongings by the wall – noncompliance will result in a score of 0 on the test.
- If you wish to leave the room during the test, you should ask permission first & turn in your test to me. Only in exceptional circumstances will I let you continue the test should you return. (So it is better to be 3 minutes late to the test, rather than ask to go to the bathroom during the test.)
- I plan to return the graded test to you on Thurs Feb 19 or Tues Feb 24.

The following questions are for your practice for the test to see how I might phrase some questions (especially multiple choice); they will not be graded and do NOT form a model for the test. Finish the homework assignments first, before working them.

PRACTICE QUESTIONS

1. Which of the following systems of equations is useful for determining the function(s) $f(t) = a + bt + ct^2$ whose graph passes through the points $(0,\ 2),\ (1,\ 8),\ (-1,\ 18)$?

(a) $\begin{aligned} a &= -2 \\ a + 4b &= -8 \\ a - b + c &= -18 \end{aligned}$ (b) $\begin{aligned} b &= 2 \\ a + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (c) $\begin{aligned} a &= 2 \\ a + 4b &= 8 \\ a - b + c &= 18 \end{aligned}$

(d) $\begin{aligned} a &= -2 \\ a + b + c &= -8 \\ a - b + c &= -18 \end{aligned}$ (e) $\begin{aligned} a &= 2 \\ a + b + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (f) $\begin{aligned} a + b &= 2 \\ a + c &= 8 \\ b - c &= 18. \end{aligned}$

2. Which of the following systems of equations is useful for determining the function(s) $f(t) = a\cos(2t) + b\sin(2t)$ that satisfy the equation $\dfrac{d^2f}{dt^2} + 2\dfrac{df}{dt} + 3f(t) = 5\cos(2t)$ for all $t \in \mathbb{R}$?

(a) $\begin{aligned} 4a - b &= 0 \\ a + 4b &= -5 \end{aligned}$ (b) $\begin{aligned} 4a + b &= 0 \\ a - 4b &= -5 \end{aligned}$ (c) $\begin{aligned} a + 4b &= 0 \\ a + 4b &= -5 \end{aligned}$ (d) $\begin{aligned} a + 4b &= 0 \\ a + b &= -5 \end{aligned}$

(e) $\begin{aligned} a + b &= 0 \\ a + 2b &= -5 \end{aligned}$ (f) $\begin{aligned} a - b &= 0 \\ a - 2b &= -5. \end{aligned}$

3. Write down the coefficient matrix and the augmented matrix (stating which is which) for the system
$$\begin{aligned} x_1 - x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 &= -1 \\ x_2 - 2x_3 &= 1. \end{aligned}$$

4. The system of linear equations
$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 1 \\ 2x_2 + x_3 &= 1 \\ x_2 + x_3 &= 1 \end{aligned}$$
has

(a) no solution (b) exactly 1 solution (c) infinitely many solutions (d) exactly 2 solutions.

5. Find the rank of $\begin{bmatrix} 0 & 1 & 5 \\ 1 & 0 & -7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) 0 (b) 1 (c) 2 (d) 3
(e) 4 (f) 4.5 (g) 5 (h) 6.

6. Which one of the following explains why $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ is <u>not</u> in reduced row-echelon form?

   (a) Column 2 contains a nonzero entry.
   (b) Column 3 contains a pivot and another nonzero entry.
   (c) Column 4 contains a pivot and another nonzero entry.
   (d) Row 1 contains a pivot and another nonzero entry.
   (e) Row 3 contains a pivot and another nonzero entry.
   (f) The number of rows does not equal the number of columns.
   (g) There is not a row of zeros in the matrix.
   (h) Trick question: the matrix is in reduced row-echelon form.

7. Find the reduced row-echelon form of $\begin{bmatrix} 2 & 4 & 6 & -8 \\ 3 & 6 & 7 & 10 \end{bmatrix}$.

   (a) $\begin{bmatrix} 1 & 2 & 3 & -4 \\ 1 & 2 & \frac{7}{3} & \frac{10}{3} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 22 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 0 & 37 \\ 0 & 0 & 1 & -11 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 0 & 29 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 2 & 3 & -4 \\ 0 & 0 & 1 & -11 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & 0 & -11 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ (h) $\begin{bmatrix} 1 & 2 & 0 & 29 \\ 0 & 0 & 1 & -11 \end{bmatrix}$.

8. If $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, then which of the following is a nontrivial (i.e., nonzero) solution of the equation $Ax = 0$? (a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (g) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (h) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

9. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the set of solutions to $Ax = 0$ geometrically?

   (a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) a plane in $\mathbb{R}^4$ (d) a plane in $\mathbb{R}^3$
   (e) a line in $\mathbb{R}^3$ (f) $\mathbb{R}^3$ (g) $\mathbb{R}^4$ (h) 3 points in $\mathbb{R}^3$.

10. If $\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 5 & k \end{array}\right]$ is the augmented matrix for a system of linear equations, then for which value(s) of $k \in \mathbb{R}$ is the system consistent?

    (a) all $k \in \mathbb{R}$ except $k = 1$ (b) $k = 1$ (c) $k = 2$ (d) $k = 4$
    (e) all $k \in \mathbb{R}$ except $k = 2$ (f) all $k \in \mathbb{R}$ except $k = 4$.

11. Justify or disprove: if a given vector b is a linear combination of the columns of a matrix A, then the equation $Ax = b$ is consistent.

12. The product $\begin{bmatrix} 1 & 3 & 5 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$ equals

(a) $3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (b) $-3\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} - 4\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (c) $4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} + 3\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(d) $-4\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} - 3\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 5 \\ 4 \end{bmatrix} + \begin{bmatrix} 10 \\ 6 \end{bmatrix} + \begin{bmatrix} 8 \\ 5 \end{bmatrix}$ (f) trick question: this product is not defined.

13. If $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$ is the augmented matrix for a system of linear equations, then the solution set of the system in vector form is:

(a) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (b) $x = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (c) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

(d) there are no solutions (e) not enough information given to answer question.

14. A system of linear equations has augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & k-1 & 2 \\ 0 & 0 & (1-k)(3k-2) & 6(1-k) \end{array}\right],$$

where $k \in \mathbb{R}$. Find all values of k such that the system does NOT have a unique solution.

(a) all $k \in [\frac{2}{3},\ 1]$ (b) $\pm\frac{2}{3}$, ± 1 only (c) $\frac{2}{3}$, 1, $\frac{4}{3}$ only (d) $\frac{4}{3}$ only (e) $\frac{2}{3}$ only

(f) 1 only (g) $\frac{2}{3}$, 1 only (h) trick question: there is a unique solution for all $k \in \mathbb{R}$.

15. Let A denote an $m \times n$ matrix and let $x \in \mathbb{R}^n$. Which of the following statements is always true?

(a) The system $Ax = 0$ is always consistent.
(b) The system $Ax = 0$ is never consistent.
(c) The system $Ax = 0$ always has nonzero solutions.
(d) If A is a 3×2 matrix, then A must have a column without a pivot.

16. Given that the reduced row-echelon form of $\begin{bmatrix} 1 & 2 & 12 \\ 0 & 1 & 5 \\ 1 & 1 & k \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & k-7 \end{bmatrix}$,

for which value(s) of k is $\begin{bmatrix} 12 \\ 5 \\ k \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$?

(a) all $k \in \mathbb{R}$ except 7 (b) $k = 0$ only (c) $k = 7$ only (d) $k = 0$ & $k = 7$ only

(e) all $k \in \mathbb{R}$ except 0 & 7 (f) all $k \in \mathbb{R}$ (g) there is no such k (h) not enough information given.

17. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{cases} \begin{bmatrix} x_1 \\ \frac{x_2^2}{x_1} \end{bmatrix} & \text{if } x_1 \neq 0 \\ \begin{bmatrix} 0 \\ x_2 \end{bmatrix} & \text{if } x_1 = 0. \end{cases}$

Show that there exist $u, v \in \mathbb{R}^2$ such that $T(u+v) \neq T(u) + T(v)$. (Hint: consider $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.)

18. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$, find $T\left(\begin{bmatrix} 5 \\ -3 \end{bmatrix} \right)$.

(a) $\begin{bmatrix} 13 \\ 7 \end{bmatrix}$ (b) $\begin{bmatrix} -7 \\ 43 \end{bmatrix}$ (c) $\begin{bmatrix} -13 \\ -7 \end{bmatrix}$ (d) $\begin{bmatrix} 10 \\ 15 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 2 \\ 23 \end{bmatrix}$.

19. Let $u,\ v \in \mathbb{R}^3$. If $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation such that $T(u) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(v) = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, find $T(2u - 3v)$.

(a) $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ (f) not enough information is given.

20. Find the matrix of the function $T : \mathbb{R}^2 \to \mathbb{R}$, where $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = x_2^2$.

(a) $\begin{bmatrix} x_1 & x_2^2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & x_2^2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & x_2 \end{bmatrix}$ (e) $\begin{bmatrix} x_2^2 \end{bmatrix}$ (f) no such matrix.

21. Suppose a line ℓ in $\mathbb{R}^2$ contains the vector $u = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$ of length 1. Find the matrix of the (orthogonal) projection map onto ℓ.

(a) $\begin{bmatrix} 0.64 & -0.48 \\ -0.48 & 0.36 \end{bmatrix}$ (b) $\begin{bmatrix} 0.48 & -0.64 \\ -0.64 & 0.36 \end{bmatrix}$ (c) $\begin{bmatrix} 0.36 & -0.48 \\ -0.48 & 0.64 \end{bmatrix}$

(d) $\begin{bmatrix} 0.48 & -0.36 \\ -0.36 & 0.64 \end{bmatrix}$ (e) $\begin{bmatrix} 0.64 & -0.36 \\ -0.36 & 0.48 \end{bmatrix}$ (f) no such matrix.

22. Find the shear matrix that transforms $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 16 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 \\ 0 & 5 \end{bmatrix}$ (f) no such shear.

23. If $A = \begin{bmatrix} k & 2 \\ -3 & 4 \end{bmatrix}$, then for which value(s) of k is A not invertible?

(a) 2 (b) $\frac{3}{2}$ (c) -2 (d) $-\frac{3}{2}$ (e) 0 (f) 0 & 2.

I plan to post the answers to the multiple-choice problems herein at my website at roughly 3:30 pm Mon Feb 16, so work the problems first and then compare with the posted answers.

You might find it beneficial to pick 8-11 of these questions and answer them in a simulated "test environment"; i.e., time yourself for 20 minutes and do not use your notes or book etc. This will allow your brain/mind practice what it will be like to answer Math 3330 questions in a test environment.

Remember: most of the test will be based on the homework questions, so finish those questions and look over your solutions to them before the test. Reread Page 1 above to see what to bring to the test.