Math 3330-002 2/10/15

Continuing from class...

Eg/ Let $A = \begin{bmatrix} 1 & 3 \\ 1 & 6 \end{bmatrix}$. Find all matrices B that commute with A.

Sol/ Want $BA = AB$ $\Rightarrow$ B must be a 2×2 matrix.

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} a+b & 3a+6b \\ c+d & 3c+6d \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+3c & b+3d \\ a+6c & b+6d \end{bmatrix}$$

equal $\Rightarrow$

$$\begin{aligned} a+b &= a+3c \\ c+d &= a+6c \\ 3a+6b &= b+3d \\ 3c+6d &= b+6d \end{aligned}$$

$$\Rightarrow \left.\begin{aligned} b-3c &= 0 \\ a+5c-d &= 0 \\ 3a+5b-3d &= 0 \end{aligned}\right\} \Rightarrow \text{solve } \left[\begin{array}{cccc|c} 0 & 1 & -3 & 0 & 0 \\ 1 & 0 & 5 & -1 & 0 \\ 3 & 5 & 0 & -3 & 0 \end{array}\right]$$

$(b - 3c = 0)$

row ops $\rightsquigarrow \left[\begin{array}{cccc|c} 1 & 0 & 5 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $\Rightarrow$ 2 free variables: c, d

$a = d - 5c$, $b = 3c$

$$\Rightarrow B = \begin{bmatrix} d-5c & 3c \\ c & d \end{bmatrix} \quad \forall\, c, d \in \mathbb{R}$$

Subeg $\left.\begin{aligned} c &= 1 \\ d &= 0 \end{aligned}\right\} \Rightarrow B = \begin{bmatrix} -5 & 3 \\ 1 & 0 \end{bmatrix}$; $\left.\begin{aligned} c &= 0 \\ d &= 1 \end{aligned}\right\} \Rightarrow B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\left.\begin{aligned} c &= 1 \\ d &= 1 \end{aligned}\right\} \Rightarrow B = \begin{bmatrix} -4 & 3 \\ 1 & 1 \end{bmatrix}$, etc.