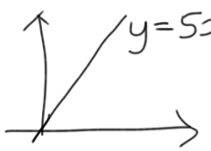
Continuing the idea of projection from class, if L is a line through the origin in $\mathbb{R}^2$ at angle θ to x-axis, then the matrix of this projection is

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} = \begin{bmatrix} u_1^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{bmatrix} \quad \text{where } u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \text{ is}$$

a vector of length 1 parallel to L.

Eg If L is line $y = 5x$, find the matrix of the projection map onto L.

Sol 1st find u.



$w = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ is a vector parallel to L, but it does not have length 1.

length of w is $\sqrt{1^2+5^2} = \sqrt{26}$.

$\Rightarrow$ take $u = \frac{1}{\sqrt{26}}\begin{bmatrix} 1 \\ 5 \end{bmatrix} \Rightarrow u_1 = 1/\sqrt{26}, \; u_2 = 5/\sqrt{26}$

$$\Rightarrow \text{matrix} = \begin{bmatrix} u_1^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{bmatrix} = \begin{bmatrix} 1/26 & 5/26 \\ 5/26 & 25/26 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 1 & 5 \\ 5 & 25 \end{bmatrix}.$$

both of these considered correct answers.

In $\mathbb{R}^3$, if L is parallel to a vector $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ of length 1, then the

matrix giving the projection is $\begin{bmatrix} u_1^2 & u_1u_2 & u_1u_3 \\ u_1u_2 & u_2^2 & u_2u_3 \\ u_1u_3 & u_2u_3 & u_3^2 \end{bmatrix}$.