You should attempt all the questions, justifying your answers, but $*$ denotes the questions that are the minimum to be attempted. Your solutions will not be collected, but you can submit questions to me for feedback and/or use my office hours for feedback. All questions are potential questions for the quizzes and tests, unless otherwise stated.

The textbook is Abstract Algebra, by D. S. Dummit & R. M. Foote, 3rd Ed., Wiley; the questions below are from this book. Another book that might be useful as a reference is Algebra, by T. W. Hungerford, Graduate Texts in Mathematics, Vol. 73, Springer. Use of other books is encouraged.

1. $*$ Read pages 1-11.
2. $*$ Do Exercise 1 on page 4.
3. $*$ Do Exercise 2 on page 4.
4. $*$ Do Exercise 3 on page 4.
5. $*$ Do Exercise 4 on page 4.
6. $*$ Do Exercise 5 on page 4.
7. $*$ Do Exercise 3 on page 11.
8. $*$ Do Exercise 6 on page 11.
9. $*$ Read pages 16-21.
10. $*$ Verify that $\{1, 2, \ldots, 9\}$ is NOT a group under multiplication modulo 10 (i.e., multiply, then reduce product modulo 10).
11. Prove that $\{e, a, a^2, a^3, \ldots, a^{n-1}\}$, where $a^n = e$, the identity, is a group under multiplication. (This group is an example of a finite cyclic group.)
12. $*$ Do Exercise 9 on page 22.
13. Do Exercise 11 on page 22.
14. Do Exercise 18 on page 22.
15. $*$ Do Exercise 22 on page 22.
16. $*$ Do Exercise 25 on page 22.
17. $*$ Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$.
18. $*$ Do Exercise 32 on page 23.
19. $*$ Do Exercise 34 on page 23.
20. $*$ Do Exercise 31 on page 23. (This question is more meaty that the preceding questions.)
21. $*$ Read pages 25-27.
22. $*$ Do Exercise 14 on page 28.
23. Do Exercise 15 on page 28.
24. $*$ Do Exercise 16 on page 28.
25. $*$ Read pages 29-32.
26. Do Exercise 1 on page 32.
27. $*$ Do Exercise 20 on page 34.
28. $*$ Read pages 34.
29. $*$ Do Exercise 1 on page 35.

30. Do Exercise 2 on page 35.
31. $*$ Do Exercise 3 on page 35.
32. $*$ Read pages 36-39.
33. Do Exercise 1 on page 36.
34. Do Exercise 3 on page 36.
35. $*$ Do Exercise 1 on page 39.
36. $*$ Do Exercise 2 on page 40.
37. Do Exercise 3 on page 40.
38. Do Exercise 8 on page 40.
39. $*$ Do Exercise 15 on page 40.
40. Do Exercise 17 on page 40.
41. $*$ Do Exercise 18 on page 40.
42. $*$ Do Exercise 22 on page 41.
43. $*$ Read pages 41-44.
44. Do Exercise 2 on page 44.
45. Do Exercise 3 on page 44.
46. $*$ Do Exercise 14 on page 45.
47. $*$ Do Exercise 15 on page 45.
48. $*$ Do Exercise 16 on page 45.
49. Read pages 46-48.
50. $*$ Do Exercise 1(a) on page 48.
51. $*$ Do Exercise 1(e) on page 48.
52. $*$ Do Exercise 2(a) on page 48.
53. Do Exercise 5 on page 48.
54. $*$ Let G be the multiplicative group of all invertible 2×2 matrices with rational entries. Show that $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ has order 4 and that $B = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ has order 3, but that AB has infinite order. On the other hand, show that the additive group $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$ contains nonzero elements a, b of infinite order such that $a + b$ has finite order.
55. $*$ Do Exercise 8 on page 48.
56. $*$ Do Exercise 10(a) on page 48.
57. Do Exercise 11 on page 48.
58. Do Exercise 12 on page 49.
59. Do Exercise 15 on page 49.
60. $*$ Read pages 49-52.
61. $*$ Do Exercise 5(a) on page 52.
62. $*$ Do Exercise 5(b) on page 52.
63. $*$ Do Exercise 11 on page 53.
64. Read pages 54-59.
65. Do Exercise 2 on page 60.

66. $*$ Do Exercise 3 on page 60.
67. $*$ Do Exercise 12 on page 60.
68. Do Exercise 24 on page 61.
69. Read pages 61-64.
70. $*$ Do Exercise 1 on page 65.
71. Do Exercise 2 on page 65.
72. $*$ Do Exercise 5 on page 65.
73. $*$ Do Exercise 6 on page 65.
74. Do Exercise 14(a)(b) on page 65.
75. $*$ Read page 905.
76. $*$ Read Appendix I.2 (starts page 907).
77. $*$ Read pages 66-71.
78. Read pages 73-85.
79. $*$ Do Exercise 7 on page 85.
80. Do Exercise 8 on page 85.
81. $*$ Do Exercise 10 on page 86.
82. Do Exercise 20 on page 87.
83. $*$ Do Exercise 36 on page 89.
84. Do Exercise 1 on page 85.
85. $*$ Do Exercise 3 on page 85.
86. $*$ Do Exercise 4 on page 85.
87. $*$ Do Exercise 5 on page 85.
88. Do Exercise 25(a) on page 88.
89. Do Exercise 27 on page 88.
90. $*$ Prove that the intersection of any collection of normal subgroups of a group G is a normal subgroup of G.
91. $*$ Do Exercise 37 on page 89.
92. $*$ Do Exercise 41 on page 89.
93. $*$ Read pages 89-95.
94. $*$ Do Exercise 1 on page 95.
95. $*$ Prove that, up to isomorphism, there are only two distinct groups of order 4, namely $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$.
96. $*$ Do Exercise 4 on page 95.
97. $*$ Do Exercise 5 on page 95.
98. Do Exercise 6 on page 95.
99. $*$ Do Exercise 12 on page 96.
100. Do Exercise 16 on page 96.
101. $*$ Read pages 97-100.
102. $*$ Let $\phi : G \to G'$ be a group homomorphism. Prove that ϕ is injective if and only if $\ker\phi = \{1\}$.

103. $*$ Do Exercise 3 on page 101.
104. $*$ Read pages 101-105.
105. $*$ Do Exercise 1 on page 106.
106. $*$ Read pages 107-110.
107. $*$ Do Exercise 1 on page 111.
108. $*$ Do Exercise 2 on page 111.
109. Do Exercise 3 on page 111.
110. $*$ Read pages 112-116.
111. $*$ Do Exercise 1 on page 116.
112. $*$ Do Exercise 2 on page 116.
113. Do Exercise 3 on page 116.
114. Do Exercise 4 on page 116.
115. $*$ Do Exercise 6 on page 116.
116. $*$ Read pages 118-120.
117. $*$ Do Exercise 1 on page 121.
118. Do Exercise 2 on page 121.
119. $*$ Do Exercise 8 on page 121.
120. $*$ Do Exercise 9 on page 121.
121. $*$ Do Exercise 14 on page 121.
122. $*$ Read pages 122-129.
123. Do Exercise 2(a) on page 130.
124. Do Exercise 3(a) on page 130.
125. $*$ Do Exercise 5 on page 130.
126. $*$ Do Exercise 6 on page 130. (Hint: $15 = |G| = 1 + 3n + 5m$; find n and m.)
127. $*$ Do Exercise 13 on page 130.
128. $*$ Read pages 133-137.
129. Do Exercise 1 on page 137.
130. Do Exercise 2 on page 137.
131. $*$ Do Exercise 6 on page 137.
132. $*$ Do Exercise 7 on page 137.
133. $*$ Do Exercise 8(a) on page 137.
134. $*$ Do Exercise 12 on page 138.
135. $*$ Do Exercise 13 on page 138.
136. $*$ Do Exercise 14 on page 138.
137. $*$ Read pages 139-146.
138. $*$ Do Exercise 13 on page 147.
139. $*$ Do Exercise 14 on page 147.
140. $*$ Do Exercise 15 on page 147.
141. $*$ Do Exercise 16 on page 147.

142. Do Exercise 17 on page 147. (Hint: see last example on page 143.)
143. Do Exercise 18 on page 147.
144. Do Exercise 19 on page 147.
145. $*$ Do Exercise 30 on page 147.
146. $*$ Read page 149.
147. $*$ Read pages 152-155.
148. Do Exercise 1 on page 156.
149. Do Exercise 7 on page 156.
150. Do Exercise 15 on page 156.
151. $*$ Do Exercise 18(a)-(c) on page 157.
152. $*$ Skim pages 158-165.
153. $*$ What are the elementary divisors of the group $\mathbb{Z}_2 \times \mathbb{Z}_9 \times \mathbb{Z}_{35}$; what are its invariant factors? Do the same for $\mathbb{Z}_{26} \times \mathbb{Z}_{42} \times \mathbb{Z}_{49} \times \mathbb{Z}_{200} \times \mathbb{Z}_{1000}$.
154. (a) $*$ How many subgroups of order 4 does $\mathbb{Z}_4 \times \mathbb{Z}_8$ have?
    (b) $*$ How many subgroups of order 25 does $\mathbb{Z}_{25} \times \mathbb{Z}_{125}$ have?
    (c) $*$ How many subgroups of order p^2 does $\mathbb{Z}_{p^2} \times \mathbb{Z}_{p^3}$ have, where p is a prime?
155. $*$ Show that a finite abelian group that is not cyclic contains a subgroup which is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$ for some prime p.
156. Do Exercise 1(a)(b) on page 165.
157. Do Exercise 2(a)-(c) on page 165.
158. Do Exercise 3(a)-(c) on page 165.
159. $*$ Do Exercise 4 on page 166.
160. $*$ Do Exercise 9 on page 166.
161. $*$ Do Exercise 10 on page 166.
162. $*$ Read pages 169-171.
163. $*$ Read pages 175-177 & Theorem 12 on page 180 .
164. $*$ Read pages 188-198.
165. $*$ Show that S_n is solvable for all $n \leq 4$, but that S_3 and S_4 are not nilpotent.
166. $*$ Do Exercise 6 on page 198. (Hint: use Theorem 8 to show $G^n \subseteq Z(G)$ for some $n \in \mathbb{Z}^+$.)
167. $*$ Read pages 215-220 & pages 222-230.
168. Do Exercise 1 on page 230.
169. Do Exercise 2 on page 230.
170. $*$ Do Exercise 3 on page 230.
171. $*$ Do Exercise 7 on page 231.
172. Do Exercise 9 on page 231.
173. $*$ Do Exercise 11 on page 231.
174. $*$ Do Exercise 28 on page 233.
175. $*$ Read pages 234-237.
176. $*$ Do Exercise 3 on page 238.

177. Do Exercise 8 on page 239.
178. $*$ Do Exercise 10(a)(c)(d) on page 238.
179. $*$ Do Exercise 11(a)(c)(d) on page 238.
180. $*$ Do Exercise 12 on page 238.
181. $*$ Read pages 239-247.
182. $*$ Do Exercise 6 on page 248.
183. Do Exercise 7 on page 248.
184. $*$ Do Exercise 10(a)(e)(f) on page 248.
185. $*$ Prove that the only ideals in $M_2(\mathbb{R})$ are 0 and $M_2(\mathbb{R})$. Verify that $\left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} : a, b \in \mathbb{R} \right\}$ is a left ideal of $M_2(\mathbb{R})$ but not a right ideal. Give a right ideal of $M_2(\mathbb{R})$ that is not a left ideal.
186. $*$ Prove that the only ideals in a field F are 0 and F. What can you say about left (resp. right) ideals in a division ring?
187. $*$ Do Exercise 16 on page 249.
188. $*$ Do Exercise 17 on page 249.
189. Suppose R is a ring in which $0 \neq 1 \in R$. Let A be a nonempty subset of R.
    (a) Prove that $\left\{ \sum_{i=1}^{m} r_i a_i : r_i \in R,\ a_i \in A,\ m \in \mathbb{Z}^+ \right\}$ is a left ideal of R.
    (b) Prove that $\left\{ \sum_{i=1}^{m} r_i a_i t_i : r_i,\ t_i \in R,\ a_i \in A,\ m \in \mathbb{Z}^+ \right\}$ is an ideal of R.
190. Do Exercise 18 on page 249.
191. $*$ Do Exercise 22 on page 249.
192. $*$ Do Exercise 34 on page 250. (Hint for (c): try an example where $I = J$.)
193. $*$ Read pages 251-254.
194. $*$ Do Exercise 4 on page 256.
195. $*$ Prove that if R is a noncommutative division ring, then $\{0\}$ is a maximal ideal. Give an example to show that the converse if false. (Hint: look back at the last dozen exercises!)
196. $*$ Do Exercise 5 on page 256.
197. Do Exercise 6 on page 256.
198. Show that the ideal $\langle x^2 \rangle$ in $\mathbb{R}[x]$ is not a prime ideal.
199. Do Exercise 7 on page 256. (Hint: prove $R[x]/\langle x \rangle \cong R$.)
200. $*$ Do Exercise 8 on page 256.
201. $*$ Do Exercise 9 on page 256.
202. $*$ Do Exercise 10 on page 257.
203. Do Exercise 19 on page 258.
204. Do Exercise 37 on page 259.
205. Do Exercise 38 on page 259.
206. $*$ Read pages 337-339 and pages 342-343.
207. If R is a ring and M is a left R-module, prove that $r0 = 0$ for all $r \in R$.
208. $*$ Do Exercise 1 on page 343.

209. Do Exercise 2 on page 343.

210. Do Exercise 3 on page 343.

211. $*$ Let R be a ring and M a left R-module. If $m \in M$, let $Rm = \{rm \ : \ r \in R\}$. Prove that Rm is a submodule of M.

212. $*$ Do Exercise 9 on page 344.

213. Let S denote the noncommutative ring consisting of finite linear combinations of words in x and y with coefficients in $\mathbb{C}$. (So S looks somewhat like the polynomial ring in that exponents are nonnegative, but there are more elements since $xy \neq yx$.) Let V denote an infinite-dimensional $\mathbb{C}$-vector space with countable basis $\{v_0, v_1, v_2, \ldots\}$. Fix $\lambda \in \mathbb{C} \setminus \{0\}$, and define $xv_i = \lambda^i v_{i+1}$ and $yv_i = v_{i+1}$ for all $i \geq 0$. Verify that V is a left S-module on which $xy - \lambda yx$ vanishes. Deduce that V is a left R-module where $R = S/\langle xy - \lambda yx \rangle$. (In fact, both S and R are $\mathbb{C}$-algebras and S is called the free $\mathbb{C}$-algebra on two indeterminates, and R is used in quantum physics and is called the coordinate ring of the quantum plane.)

214. $*$ Read pages 260-264 and page 707.

215. Let $R = \mathbb{C}[x, \ y]$ and $D = R \setminus \langle y \rangle$. Show that $D^{-1}R \ncong S[y]$, where S denotes the ring of rational functions in x (c.f., Example 4 on page 264).

216. Complete the student feedback survey online. (It needs to be completed by 11 pm Fri May 2.) Thank you!

END HOMEWORK