Math 5333

Sec 5C

Here is a proof of (e) => (b) from our last theorem in Section 5C. We proved the rest of that result in class.

$\dim(V)<\infty$, $T \in \mathcal{L}(V)$, $\lambda_1, \ldots, \lambda_m$ are all the distinct evals of T.

(b) V has a basis $\mathcal{B}'$ that consists of evecs of T.

(e) $\dim(V) = \sum_{i=1}^{m} \dim(E_{\lambda_i})$.

Proof of (e) $\Rightarrow$ (b)

Let $E = \sum_{i=1}^{m} E_{\lambda_i}$. By last prop, $E = \bigoplus_{i=1}^{m} E_{\lambda_i}$.

By dim formula, $\dim(E) = \sum_{i=1}^{m} \dim(E_{\lambda_i}) \overset{(e)}{=} \dim(V)$

$\dim(V)<\infty \Rightarrow E = V$ $(\Rightarrow (d))$ ①

Let $\mathcal{B}_i$ = basis of E_{λ_i} $\forall\, i$ and let $\mathcal{B}' = \bigcup_{i=1}^{m} \mathcal{B}_i$.

By (e), $|\mathcal{B}'| = \dim(V)$ ②

By ①, $\mathcal{B}'$ spans V. ③

② + ③ $\Rightarrow$ $\mathcal{B}'$ = basis of V. $\Rightarrow$ (b). ■