FINAL TEST will be in PKH 319 on Tues Dec 15, 5:30-8:00 pm

- See above for date, time and location of FINAL TEST; it is worth 38% of your course grade.
- As stated in the course information sheet, **any student not obtaining a positive score on the FINAL TEST will not pass this class.**
- The material to be covered on the test is the same as that covered in **Chapters 1-3, 5-8, and bilinear forms**.
  (Homework is at my website: http://www.uta.edu/math/vancliff/T/F15/hwk5333.html .)
- My office hours are Tues & Thurs 4:45-5:20 pm in PKH 462 until Tues Dec 8 inclusive. In addition, I will be in my office (PKH 462) on Saturday Dec 12 from approximately 2:15 pm until 4:15 pm, and again on Monday Dec 15 from approximately 2:30 pm until 4:00 pm.
- When I write a test, I look over the lecture notes and homework which have already been assigned, and any previous tests, and use them to model about 85% of the test problems (and most of them are fair game). You should expect 11-14 questions in total.
- You should know the definitions of the concepts (and examples thereof) that arise frequently in class; the theorems etc that are used frequently or which have a name; how to apply the theorems to examples and at least one method for doing each homework question. Learn any formulae taught in class. Skimming over short proofs (at most 6 or so lines) from lecture could also be helpful.
- A good way to review is to go over the homework problems you have not already done & make sure you understand all the homework well by 48 hours prior to the test. Consider adopting one or more of the study techniques linked at my website.
- You should look over the previous tests; the tests and their solutions are posted online, with links to them provided at the homework website above (use password "math5333" to open the files). Look over the examples on the class handouts; see my website for them.
- It is your responsibility to be on time.
- Try to keep your eyes on your own work during the test.
- BRING YOUR MYMAV ID CARD WITH YOU. All other nontest items will have to be put by the side walls or by my desk prior to the start of the test; this includes cell phones & tablets, which need to be switched off & in with your belongings by the wall – noncompliance will result in a score of 0 on the test.
- If you wish to leave the room during the test, you should ask permission first & turn in your test to me. Only in exceptional circumstances will I let you continue the test should you return. (So it is better to be 3 minutes late to the test, rather than ask to go to the bathroom during the test.)
- I hope to have the test graded by Dec 21.
- I would like to take this opportunity to remind y'all to complete the student **feedback survey** for all your classes by **11 pm Wed Dec 9**; see your mymav email for the link. Thank you!

In the following, $\mathbb{F}$ denotes $\mathbb{R}$ or $\mathbb{C}$.

In order to solve systems of linear equations with coefficients in $\mathbb{F}$, we consider $\mathbb{F}^n$ and $m \times n$ matrices with entries in $\mathbb{F}$. This leads us to the notion of vector space over $\mathbb{F}$, and linear maps from one vector space to another. To eliminate redundancy in a vector space and to find "simple" ways to describe vector spaces, we consider linear (in)dependence, spanning sets, bases and ways to write a vector space as a (direct) sum of subspaces. To solve differential equations, one sometimes needs to consider infinite-dimensional vector spaces, which are studied in detail in functional analysis.

We represent a linear map from one finite-dimensional vector space to another by a matrix, and use matrix multiplication to compute the composition of two linear maps.

We explored the notion of the dual V' of a vector space V, which is simply all the linear maps from V to the field, and is itself a vector space. The dual is used in differential geometry (tangent space vs cotangent space) and, hence, in the subject of special, and general, relativity. It even arises in the subject of classical and quantum physics. If there is a linear map T from one vector space to another, then T has a "dual" map that we called T'. If the vector spaces are finite-dimensional, and if matrices are used with respect to bases and dual bases, then the matrix of T' is the transpose of the matrix of T.

If the vector space is finite-dimensional, then a linear map from it to itself is represented by a square matrix. In this case, we use the notion of eigenvector and eigenvalue to better understand the linear map. The notions of eigenvector and eigenvalue lead us to the notions of eigenspaces and invariant subspaces. If the linear map provides an eigenbasis for the vector space, then there exists a diagonal matrix that represents the linear map.

On some vector spaces, an inner product can be defined. In this case, we can consider the notions of norm and orthogonality, and these lead to the notion of orthonormal vectors. Such vectors can be constructed from linearly indepenent vectors by using the Gram-Schmidt orthogonalization procedure (GSOP). Orthonormal vectors are automatically linearly independent, and it is often convenient to use an orthonormal basis of our subspace or vector space. Using orthogonal complements, one can find a vector in a subspace that is as "close as possible" to a given generic vector in our inner-product space. This latter type of situation is useful in statistics, engineering, geology and some other applied fields.

Related to the idea of inner product is the notion of a bilinear form. Indeed, real inner products are special cases of bilinear forms. Symmetric bilinear forms on finite-dimensional vector spaces, quadratic forms and symmetric matrices are interchangeable. Although, in our class, the symmetric matrices used in this setting have scalar entries, one of the main applications of these concepts is in the area of photogrammetry, where the symmetric matrices that are used have polynomial entries.

Using inner products on vector spaces, one can define the adjoint of a linear map, which itself is a linear map (but we had to prove various technical results first in order to show that the adjoint can indeed be defined). Any linear map that is equal to its adjoint is called "self-adjoint", while any linear map that commutes with its adjoint is called "normal". Two of the three main theorems of this semester are the **complex** and **real spectral theorems** that concern representing a normal, or

self-adjoint, linear map T as a diagonal matrix with respect to a basis of orthonormal eigenvectors of T.

If a linear operator T is neither self-adjoint nor normal, then T need not yield an eigenbasis; without an eigenbasis, there is no diagonal matrix that represents T. This situation was our focus of study in Chapter 8, which ended with the Jordan normal (canonical) form (JNF) of a linear operator. If the JNF is not diagonal, then the operator does not have an eigenbasis, so generalized eigenvectors are used to find a basis. In this setting, starting with a matrix A and changing to the Jordan normal form J uses the formula $J = P^{-1}AP$, where the columns of the matrix P are given by the new basis that consists of eigenvectors and generalized eigenvectors of A.

We also reconsidered the real spectral theorem again and used the Gram-Schmidt orthogonalization procedure (GSOP) on each eigenspace in turn (with the dot product) to find a change-of-basis matrix P for a real symmetric matrix A such that the columns of P are orthonormal eigenvectors of A (so P is an orthogonal matrix). In this case, the formula above becomes $J = P^{-1}AP = P^TAP$, since $P^{-1} = P^T \equiv$ the transpose of P.

At this stage of the semester, we doubled back and looked at products of vector spaces in Chapter 3. This topic is most useful for identifying if a vector space can be written as a product of "smaller" vector spaces, but we did not explore this idea much. However, in the case where the vector spaces V_i in the product $V_1 \times V_2 \times ... \times V_m$ are finite-dimensional and are subspaces of a vector space V, the product can be used to prove a result that uses dimension to determine if the sum of the V_i is a direct sum.

The main results of the semester were the real spectral theorem, the complex spectral theorem and the existence of the Jordan normal form (& how to find it).