FINAL TEST will be in PKH 105 on Tues Dec 15, 2:00-4:30 pm

- See above box for date, time and location of Final Test; it is worth 38% of your course grade.
- As stated in the course information sheet, **any student not obtaining a positive score on the FINAL TEST will not pass this class.**
- The material to be covered on the test is the same as that covered on the homework through the end of Section 8.1 inclusive.
  (Homework is at my website: http://www.uta.edu/math/vancliff/T/F15/hwk3330.html .)
- My office hours are Tues & Thurs 3:20-4:00 pm in PKH 105 or PKH 462 until Tues Dec 8 inclusive. In addition, I will be in my office (PKH 462) on Saturday Dec 12 from approximately 2:15 pm until 4:15 pm, and again on Monday Dec 15 from approximately 2:30 pm until 4:00 pm. Also, you have access to the math clinic during the hours it is open.
- This test will be mainly (but not entirely) multiple choice; you do NOT need to bring a scantron form. There will be several choices of answer per multiple-choice question and, for each, only one answer will be the correct one. **For show-work questions, work and reasoning must be shown to earn credit.**
- You should do rough work on the test or on paper provided by me. No calculators, notes, cards, phones, tablets, computers or any electronic devices are allowed.

BRING YOUR MYMAV ID CARD WITH YOU.

- When I write a test, I look over the lecture notes and homework that have already been assigned, and the previous tests, and use them to model about 85% of the test problems (and most of them are fair game). You should expect 36-40 questions in total.
- A good way to review is to go over the homework problems you have not already done & make sure you understand all the homework well by 48 hours prior to the test. You should also go over the quiz and Tests 1-3; those tests and their solutions are posted online at my website (use password "math3330" to open each file); see http://www.uta.edu/math/vancliff/T/F15/3330qs.pdf , uta.edu/math/vancliff/T/F15/3330t1s.pdf , uta.edu/math/vancliff/T/F15/3330t2s.pdf & http://www.uta.edu/math/vancliff/T/F15/3330t3s.pdf . In addition, this information sheet provides some practice problems that are provided to help you study if you have finished all the homework questions. These practice questions do NOT form a model for the test. These questions are intended only to help you identify any gaps in your understanding. In the last 24 hours before the test, reread ALL the homework problems, skim through the lecture notes, & go over these practice questions again. Consider adopting one or more of the study techniques linked at my website. Make sure you learn any formulae taught in class & know some basic formulae such as pythagorean theorem & quadratic formula.
- It is your responsibility to be on time. Try to keep your eyes on your own work during the test.
- All nontest items will have to be put by the side walls or by my desk prior to the start of the test; this includes all electronic devices such as cell phones or tablets, which need to be switched off and in with your belongings by the wall – noncompliance will result in a score of 0 on the test.
- If you wish to leave the room during the test, you should ask permission first & turn in your test to me. Only in exceptional circumstances will I let you continue the test should you return. (So it is better to be 3 minutes late to the test, rather than ask to go to the bathroom during the test.)
- I hope to have the test graded by Dec 21.

The following questions are for your practice for the test to see how I might phrase some questions (especially multiple choice); they will not be graded and do NOT form a model for the test. Finish the homework assignments first, before working them.

PRACTICE QUESTIONS

1. Which of the following systems of equations is useful for determining the function(s) $f(t) = a + bt + ct^2$ whose graph passes through the points $(0,\ 2),\ (1,\ 8),\ (-1,\ 18)$?

(a) $\begin{aligned} a &= -2 \\ a + 4b &= -8 \\ a - b + c &= -18 \end{aligned}$ (b) $\begin{aligned} b &= 2 \\ a + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (c) $\begin{aligned} a &= 2 \\ a + 4b &= 8 \\ a - b + c &= 18 \end{aligned}$

(d) $\begin{aligned} a &= -2 \\ a + b + c &= -8 \\ a - b + c &= -18 \end{aligned}$ (e) $\begin{aligned} a &= 2 \\ a + b + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (f) $\begin{aligned} a + b &= 2 \\ a + c &= 8 \\ b - c &= 18. \end{aligned}$

2. Which of the following systems of equations is useful for determining the function(s) $f(t) = a\cos(2t) + b\sin(2t)$ that satisfy the equation $\dfrac{d^2f}{dt^2} + 2\dfrac{df}{dt} + 3f(t) = 5\cos(2t)$ for all $t \in \mathbb{R}$?

(a) $\begin{aligned} 4a - b &= 0 \\ a + 4b &= -5 \end{aligned}$ (b) $\begin{aligned} 4a + b &= 0 \\ a - 4b &= -5 \end{aligned}$ (c) $\begin{aligned} a + 4b &= 0 \\ a + 4b &= -5 \end{aligned}$ (d) $\begin{aligned} a + 4b &= 0 \\ a + b &= -5 \end{aligned}$

(e) $\begin{aligned} a + b &= 0 \\ a + 2b &= -5 \end{aligned}$ (f) $\begin{aligned} a - b &= 0 \\ a - 2b &= -5. \end{aligned}$

3. Write down the coefficient matrix and the augmented matrix (stating which is which) for the system
$$\begin{aligned} x_1 - x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 &= -1 \\ x_2 - 2x_3 &= 1. \end{aligned}$$

4. The system of linear equations
$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 1 \\ 2x_2 + x_3 &= 1 \\ x_2 + x_3 &= 1 \end{aligned}$$
has

(a) no solution (b) exactly 1 solution (c) infinitely many solutions (d) exactly 2 solutions.

5. Find the rank of $\begin{bmatrix} 0 & 1 & 5 \\ 1 & 0 & -7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) 0 (b) 1 (c) 2 (d) 3
(e) 4 (f) 4.5 (g) 5 (h) 6.

6. Which one of the following explains why $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ is <u>not</u> in reduced row-echelon form?

   (a) Column 2 contains a nonzero entry.
   (b) Column 3 contains a pivot and another nonzero entry.
   (c) Column 4 contains a pivot and another nonzero entry.
   (d) Row 1 contains a pivot and another nonzero entry.
   (e) Row 3 contains a pivot and another nonzero entry.
   (f) The number of rows does not equal the number of columns.
   (g) There is not a row of zeros in the matrix.
   (h) Trick question: the matrix is in reduced row-echelon form.

7. Find the reduced row-echelon form of $\begin{bmatrix} 2 & 4 & 6 & -8 \\ 3 & 6 & 7 & 10 \end{bmatrix}$.

   (a) $\begin{bmatrix} 1 & 2 & 3 & -4 \\ 1 & 2 & \frac{7}{3} & \frac{10}{3} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 22 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 0 & 37 \\ 0 & 0 & 1 & -11 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 0 & 29 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 2 & 3 & -4 \\ 0 & 0 & 1 & -11 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & 0 & -11 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ (h) $\begin{bmatrix} 1 & 2 & 0 & 29 \\ 0 & 0 & 1 & -11 \end{bmatrix}$.

8. If $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, then which of the following is a nontrivial (i.e., nonzero) solution of the equation $Ax = 0$? (a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (g) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (h) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

9. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the set of solutions to $Ax = 0$ geometrically?

   (a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) a plane in $\mathbb{R}^4$ (d) a plane in $\mathbb{R}^3$
   (e) a line in $\mathbb{R}^3$ (f) $\mathbb{R}^3$ (g) $\mathbb{R}^4$ (h) 3 points in $\mathbb{R}^3$.

10. If $\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 5 & k \end{array}\right]$ is the augmented matrix for a system of linear equations, then for which value(s) of $k \in \mathbb{R}$ is the system consistent?

    (a) all $k \in \mathbb{R}$ except $k = 1$ (b) $k = 1$ (c) $k = 2$ (d) $k = 4$
    (e) all $k \in \mathbb{R}$ except $k = 2$ (f) all $k \in \mathbb{R}$ except $k = 4$.

11. Justify or disprove: if a given vector b is a linear combination of the columns of a matrix A, then the equation $Ax = b$ is consistent.

12. The product $\begin{bmatrix} 1 & 3 & 5 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$ equals

(a) $3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (b) $-3\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} - 4\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (c) $4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} + 3\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(d) $-4\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 7\begin{bmatrix} 3 \\ -1 \end{bmatrix} - 3\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 5 \\ 4 \end{bmatrix} + \begin{bmatrix} 10 \\ 6 \end{bmatrix} + \begin{bmatrix} 8 \\ 5 \end{bmatrix}$ (f) trick question: this product is not defined.

13. If $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$ is the augmented matrix for a system of linear equations, then the solution set of the system in vector form is:

(a) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (b) $x = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (c) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

(d) there are no solutions (e) not enough information given to answer question.

14. A system of linear equations has augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & k-1 & 2 \\ 0 & 0 & (1-k)(3k-2) & 6(1-k) \end{array}\right],$$

where $k \in \mathbb{R}$. Find all values of k such that the system does NOT have a unique solution.

(a) all $k \in [\frac{2}{3},\ 1]$ (b) $\pm\frac{2}{3}$, ± 1 only (c) $\frac{2}{3}$, 1, $\frac{4}{3}$ only (d) $\frac{4}{3}$ only (e) $\frac{2}{3}$ only

(f) 1 only (g) $\frac{2}{3}$, 1 only (h) trick question: there is a unique solution for all $k \in \mathbb{R}$.

15. Let A denote an $m \times n$ matrix and let $x \in \mathbb{R}^n$. Which of the following statements is always true?

(a) The system $Ax = 0$ is always consistent.
(b) The system $Ax = 0$ is never consistent.
(c) The system $Ax = 0$ always has nonzero solutions.
(d) If A is a 3×2 matrix, then A must have a column without a pivot.

16. Given that the reduced row-echelon form of $\begin{bmatrix} 1 & 2 & 12 \\ 0 & 1 & 5 \\ 1 & 1 & k \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & k-7 \end{bmatrix}$,

for which value(s) of k is $\begin{bmatrix} 12 \\ 5 \\ k \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$?

(a) all $k \in \mathbb{R}$ except 7 (b) $k = 0$ only (c) $k = 7$ only (d) $k = 0$ & $k = 7$ only

(e) all $k \in \mathbb{R}$ except 0 & 7 (f) all $k \in \mathbb{R}$ (g) there is no such k (h) not enough information given.

17. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{cases} \begin{bmatrix} x_1 \\ \frac{x_2^2}{x_1} \end{bmatrix} & \text{if } x_1 \neq 0 \\ \begin{bmatrix} 0 \\ x_2 \end{bmatrix} & \text{if } x_1 = 0. \end{cases}$

Show that there exist $u, v \in \mathbb{R}^2$ such that $T(u+v) \neq T(u) + T(v)$. (Hint: consider $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.)

18. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$, find $T\left(\begin{bmatrix} 5 \\ -3 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} 13 \\ 7 \end{bmatrix}$ (b) $\begin{bmatrix} -7 \\ 43 \end{bmatrix}$ (c) $\begin{bmatrix} -13 \\ -7 \end{bmatrix}$ (d) $\begin{bmatrix} 10 \\ 15 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 2 \\ 23 \end{bmatrix}$.

19. Let $u,\ v \in \mathbb{R}^3$. If $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation such that $T(u) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(v) = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, find $T(2u - 3v)$.

(a) $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ (f) not enough information is given.

20. Find the matrix of the function $T : \mathbb{R}^2 \to \mathbb{R}$, where $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_2^2$.

(a) $\begin{bmatrix} x_1 & x_2^2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & x_2^2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & x_2 \end{bmatrix}$ (e) $\begin{bmatrix} x_2^2 \end{bmatrix}$ (f) no such matrix.

21. Suppose a line ℓ in $\mathbb{R}^2$ contains the vector $u = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$ of length 1. Find the matrix of the (orthogonal) projection map onto ℓ.

(a) $\begin{bmatrix} 0.64 & -0.48 \\ -0.48 & 0.36 \end{bmatrix}$ (b) $\begin{bmatrix} 0.48 & -0.64 \\ -0.64 & 0.36 \end{bmatrix}$ (c) $\begin{bmatrix} 0.36 & -0.48 \\ -0.48 & 0.64 \end{bmatrix}$

(d) $\begin{bmatrix} 0.48 & -0.36 \\ -0.36 & 0.64 \end{bmatrix}$ (e) $\begin{bmatrix} 0.64 & -0.36 \\ -0.36 & 0.48 \end{bmatrix}$ (f) no such matrix.

22. Find the shear matrix that transforms $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 16 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 \\ 0 & 5 \end{bmatrix}$ (f) no such shear.

23. If $A = \begin{bmatrix} k & 2 \\ -3 & 4 \end{bmatrix}$, then for which value(s) of k is A not invertible?

(a) 2 (b) $\frac{3}{2}$ (c) -2 (d) $-\frac{3}{2}$ (e) 0 (f) 0 & 2.

24. Compute $\begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. (a) $\begin{bmatrix} 2 & 2 & 7 \\ -2 & 4 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ 2 & 6 \\ -3 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & -2 \\ 2 & 4 \\ 7 & 10 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} -1 & 2 & -3 \\ -1 & 6 & -5 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & -2 \\ 6 & 0 \end{bmatrix}$ (g) cannot be multiplied.

25. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 2 & 1 & 4 \end{bmatrix}$, then which of the following is the second row of A^{-1}?

(a) $1 \;\; 1 \;\; 1$ (b) $0 \;\; 0 \;\; 1$ (c) $4 \;\; -2 \;\; -3$ (d) $1 \;\; 2 \;\; -3$ (e) $2 \;\; 3 \;\; 4$ (f) $4 \;\; 5 \;\; -6$
(g) $0 \;\; -2 \;\; 1$ (h) $0 \;\; 2 \;\; 1$ (i) A^{-1} does not exist.

26. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 7 \\ 10 \\ 9 \end{bmatrix}$. Describe $\text{span}(v_1, v_2, v_3) = \mathbb{R}v_1 + \mathbb{R}v_2 + \mathbb{R}v_3$.

(a) a point (b) a line (c) a plane (d) $\mathbb{R}^3$ (e) exactly 3 points
(f) exactly 3 vectors emerging from the origin in $\mathbb{R}^3$.

27. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the kernel of A geometrically?

(a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) 3 points in $\mathbb{R}^3$
(d) a line in $\mathbb{R}^3$ (e) a plane in $\mathbb{R}^3$ (f) a plane in $\mathbb{R}^9$ (g) $\mathbb{R}^3$ (h) $\mathbb{R}^4$.

28. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the image of A geometrically?

(a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) 3 points in $\mathbb{R}^3$
(d) a line in $\mathbb{R}^3$ (e) a plane in $\mathbb{R}^3$ (f) a plane in $\mathbb{R}^9$ (g) $\mathbb{R}^3$ (h) $\mathbb{R}^4$.

29. The set $U = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ 5 \end{bmatrix} : x_1, x_2 \in \mathbb{R} \right\}$ is not a subspace of $\mathbb{R}^3$ because

(a) U is not a subset of $\mathbb{R}^3$ (b) U is empty (c) U contains $\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$ (d) U contains the origin
(e) U does not contain the origin (f) trick question: U is a subspace of $\mathbb{R}^3$.

30. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, $v_3 = \begin{bmatrix} 9 \\ 2 \\ 33 \end{bmatrix}$. Given that $\begin{bmatrix} 5 \\ -7 \\ 1 \end{bmatrix}$ is in the kernel of the matrix $A = \begin{bmatrix} 1 & 2 & 9 \\ 1 & 1 & 2 \\ -1 & 4 & 33 \end{bmatrix}$, find a nontrivial relation among v_1, v_2 and v_3.

Show some work; write clearly.

31. Which of the following sets of vectors is linearly independent?

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ (f) none of these.

32. Which of the sets in the previous question is a basis for $\mathbb{R}^3$? (a) (b) (c) (d) (e) (f) none.

33. Define (i) subspace of $\mathbb{R}^n$ (ii) span of m vectors in $\mathbb{R}^n$ (iii) kernel of a linear transformation (iv) image of a linear transformation (v) basis of a subspace (vi) dimension of a subspace (vii) the coordinate vector of a vector with respect to a basis $\mathcal{B}$.

34. The set $W = \left\{ \begin{bmatrix} d \\ c \\ b \\ a \end{bmatrix} : a, b, c, d \in \mathbb{R} \text{ and } a + d = b + c \right\}$ is a subspace of $\mathbb{R}^4$. Find the dimension of W.

(a) 7 (b) 6 (c) 5 (d) 4 (e) 3 (f) 2 (g) 1 (h) 0.

35. You are given a matrix A and its reduced row-echelon form:

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 & 0 & -1 & 4 \\ 1 & 1 & 3 & 0 & 0 & 2 & 3 \\ 1 & 1 & 3 & 1 & 0 & 1 & 7 \\ 1 & 1 & 3 & 1 & 1 & 4 & 6 \\ 1 & 1 & 3 & 1 & 1 & 4 & 6 \\ 1 & 1 & 3 & 1 & 1 & 4 & 6 \end{bmatrix} \qquad \text{rref}(A) = \begin{bmatrix} 1 & 0 & 3 & 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The matrix A represents a linear transformation T where $T(x) = Ax$ and....

(a) $T : \mathbb{R}^{13} \to \mathbb{R}^6$ (b) $T : \mathbb{R}^6 \to \mathbb{R}^{13}$ (c) $T : \mathbb{R}^7 \to \mathbb{R}^6$ (d) $T : \mathbb{R}^{42} \to \mathbb{R}^{42}$

(e) $T : \mathbb{R}^6 \to \mathbb{R}^7$ (f) $T : \mathbb{R}^7 \to \mathbb{R}^{13}$ (g) $T : \mathbb{R}^{13} \to \mathbb{R}^7$ (h) not enough information.

36. Find the dimension of the image of the matrix A given in Question 35.

(a) 6 (b) 5 (c) 4 (d) 3 (e) 2 (f) 1 (g) 0 (h) not enough information.

37. Find a basis for the image of the matrix A given in Question 35.

(a) Rows 1, 2, 4 of rref(A) (b) rows 1-4 of rref(A)

(c) rows 1, 2, 4, 5 of A (d) all the rows of A

(e) columns 1, 2, 4, 5 of rref(A) (f) all the columns of rref(A)

(g) columns 1, 2, 4, 5 of A (h) all the columns of A.

38. Using the matrix A from Question 35 and a vector x, how many free variables has the homogeneous system $Ax = 0$?

(a) 7 (b) 6 (c) 5 (d) 4 (e) 3 (f) 2 (g) 1 (h) 0.

39. Find the dimension of the kernel of the matrix A given in Question 35.

(a) 7 (b) 6 (c) 5 (d) 4 (e) 3 (f) 2 (g) 1 (h) 0.

40. Find a basis for the kernel of the matrix A given in Question 35; show all work.

41. Let $A = \begin{bmatrix} -3 & 10 \\ -3 & 8 \end{bmatrix}$, $v_1 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Using $\mathcal{B} = \{v_1,\ v_2\}$ as a basis for $\mathbb{R}^2$, find $[Av_2]_{\mathcal{B}}$.

(a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (f) not enough information.

42. Let $T(x) = Ax$ for all $x \in \mathbb{R}^2$, where A is the 2×2 matrix from Question 41. Find the matrix of T with respect to the basis $\mathcal{B}$ given in Question 41.

(a) $\begin{bmatrix} 2 & 10 \\ 0 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 10 \\ -3 & 8 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 8 \\ -3 & 10 \end{bmatrix}$ (e) $\begin{bmatrix} 8 & -10 \\ 3 & -3 \end{bmatrix}$

(f) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (h) $\begin{bmatrix} 2 & -3 \\ 0 & -3 \end{bmatrix}$.

43. Find a basis for the real linear space $V = \left\{ \begin{bmatrix} a & a \\ b & b \\ c & c \end{bmatrix} \in \mathbb{R}^{3\times 2} : a, b, c \in \mathbb{R} \right\}$.

(a) $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$ (f) $\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ (g) V has a basis, but it is infinite.

44. Let $A,\ B \in \mathbb{R}^{n\times n}$. We define a map $T : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$ by $T(M) = AMB$ for all $M \in \mathbb{R}^{n\times n}$. Justify whether or not T is a linear transformation. Show all work.

45. Let $T : P_2 \to \mathbb{R}^2$ be given by $T(f) = \begin{bmatrix} 4f(1) \\ f'(4) \end{bmatrix}$, where f' denotes the derivative of the polynomial f; i.e., $T(f) = \begin{bmatrix} 4f(t)\big|_{t=1} \\ \frac{df}{dt}\big|_{t=4} \end{bmatrix}$. You may assume that T is a linear transformation. Using the basis $\{t^2, t, 1\}$ for P_2 and the basis $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^2$, find the matrix of T with respect to these bases.

(a) $\begin{bmatrix} 4 & 4 & 4 \\ 8 & 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 4 & 4 \\ 8 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 8 \\ 4 & 1 \\ 4 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 0 \\ 4 & 1 \\ 4 & 8 \end{bmatrix}$ (e) $\begin{bmatrix} 4 & 4 & 4 \\ 0 & 1 & 8 \end{bmatrix}$ (f) $\begin{bmatrix} 4 & 4 & 4 \\ 8 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

46. Using the basis $\mathcal{B} = \{t^3, t^2, t, 1\}$ for P_3, find $[3t^3 + 2t^2]_{\mathcal{B}}$.

(a) $\begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 3 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 0 \\ 0 \\ 2 \\ 3 \end{bmatrix}$ (g) $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ (h) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$.

47. Let V be the plane, $V = \mathbb{R}v_1 + \mathbb{R}v_2$ in $\mathbb{R}^3$, where $v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

Let $T : V \to V$ be a linear transformation such that $T(v_1) = v_2 - v_1$ and $T(v_2) = -3v_1$. Find the matrix of T with respect to the basis $\{v_1, v_2\}$ for V.

(a) $\begin{bmatrix} -1 & -3 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & -1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 1 \\ -3 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}$ (f) $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$ (g) $\begin{bmatrix} -2 & -3 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

48. Let $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and let $\mathcal{E} = \{e_1, e_2, e_3\}$ be the standard basis of $\mathbb{R}^3$, and assume $\mathcal{B} = \{-e_2, e_1, e_1 + e_3\}$ is another basis of $\mathbb{R}^3$. Find the matrix that changes basis from $\mathcal{B}$ to $\mathcal{E}$.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (f) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (g) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ (h) $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$.

49. Let u_1, u_2, u_3 denote orthonormal vectors in $\mathbb{R}^{10}$. Find the length of the vector $4u_1 - 3u_2 + 5u_3$.

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 3 (d) $3\sqrt{5}$ (e) 12 (f) 10 (g) 0 (h) $5\sqrt{2}$.

50. Which of the following sets of vectors form an orthonormal set?

(a) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ -\frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ \frac{2}{3} \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$ (f) none of them.

51. Use the Gram-Schmidt orthogonalization procedure to produce an orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$.

(a) $\left\{ \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ \frac{\sqrt{3}}{\sqrt{5}} \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{2}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \right\}$ (f) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{2}{\sqrt{5}} \\ 0 \\ -\frac{1}{\sqrt{5}} \end{bmatrix} \right\}$ (g) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (h) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{bmatrix} \right\}$.

52. Which one of the following matrices is orthogonal?

(a) $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{3} & \frac{1}{2} & -1 \\ -\frac{1}{3} & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (f) $\begin{bmatrix} \frac{2}{3} & \frac{2}{3} & 1 \\ -\frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & -\frac{1}{3} & 0 \end{bmatrix}$ (g) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$.

53. Which one of the following matrices is symmetric?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 9 & 6 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 8 \\ 8 & 5 \end{bmatrix}$ (f) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

54. Find $C^T + AB$, where $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 0 & -2 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 2 & 9 \end{bmatrix}$.

(a) $\begin{bmatrix} 4 & -3 \\ 0 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ 0 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & -1 \\ -2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$ (f) $\begin{bmatrix} 4 & -2 \\ -1 & 5 \end{bmatrix}$

(g) $\begin{bmatrix} 4 & 0 \\ -3 & -4 \end{bmatrix}$ (h) $\begin{bmatrix} 4 & -2 \\ -1 & 7 \end{bmatrix}$ (i) cannot be computed.

55. Find the determinant of $\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$.

(a) -15 (b) -1 (c) 1 (d) 0 (e) 2 (f) 3 (g) 5 (h) 7 (i) 9.

56. If A, B and C are $n \times n$ matrices such that $\det(A) = 5$, $\det(B) = 2$, $\det(C) = 4$, find $\det(AB^{-1}C^T)$.

(a) -4 (b) -2 (c) 0 (d) 2 (e) 4 (f) 6 (g) 8 (h) 10 (i) 12.

57. If A is a 5×5 matrix, find $\det(2A)$.

(a) $128\det(A)$ (b) $32\det(A)$ (c) $8\det(A)$ (d) $2\det(A)$ (e) 0
(f) $-2\det(A)$ (g) $-8\det(A)$ (h) $-32\det(A)$ (i) $-128\det(A)$.

58. Suppose $A = \begin{bmatrix} e^{697} & 6^{8^{100}} & 2^{-4000} \\ 9^{508} & 7^{1800} & \sqrt{3} \\ -\pi^{792} & 1 & 1 \end{bmatrix}$ is the coefficient matrix of the linear system $Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

Assuming $\det(A) \neq 0$ (so that the solution x is unique), use Cramer's Rule to find the first coordinate of x as a function of $\det(A)$.

(a) $\frac{7^{1800} - \sqrt{3}}{\det(A)}$ (b) $\frac{7^{1800} + \sqrt{3}}{\det(A)}$ (c) $\frac{6^{8^{100}} - 2^{-4000}}{\det(A)}$ (d) $\frac{6^{8^{100}} + 2^{-4000}}{\det(A)}$ (e) $\frac{e^{697} - 9^{508}}{\det(A)}$ (f) $\frac{e^{697} + 9^{508}}{\det(A)}$.

59. Let A denote an $n \times n$ matrix. Give the <u>definition</u> of an eigenvalue λ of A.

60. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -3 \\ 0 & 3 & 5 \end{bmatrix}$. The matrix A has an eigenvector $v = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ associated to an eigenvalue λ; find λ. (Hint: the fastest way to do this question does not use any polynomials.)

(a) 0 (b) -1 (c) 1 (d) -2 (e) 2 (f) -3 (g) 3 (h) -4 (i) 4.

61. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ has an eigenvector $v \in \mathbb{R}^3$ associated to an eigenvalue $\lambda = 3$; find $T^4(v)$.

(a) $243v$ (b) $9v$ (c) $81v$ (d) $99v$ (e) $3v$ (f) $27v$ (g) not enough information.

62. Find the characteristic polynomial of $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.

(a) $1 - x$ (b) $x^2 + 4x + 1$ (c) $x^2 + 4x - 1$ (d) $x^2 - 4x + 1$ (e) $x^2 - 4x - 1$
(f) $x^2 - 4x + 5$ (g) $x^2 + 4x - 5$ (h) $x^2 - 4x - 5$ (i) $x^2 + 4x + 5$.

63. Find all the eigenvalues of $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 5 \\ 0 & -6 & 8 \end{bmatrix}$.

(a) 1, 2 & 3 only (b) 2 & 3 only (c) 2, -3 & 8 only (d) 1 & 3 only (e) 1 & 2 only
(f) 1 only (g) 2 only (h) 3 only (i) A has no eigenvalues in $\mathbb{R}$.

64. The matrix $A = \begin{bmatrix} 0 & 1 & -3 \\ -3 & 4 & -3 \\ 0 & 0 & 3 \end{bmatrix}$ has eigenvalue 3. Find the eigenspace of the eigenvalue 3. Show all work and write clearly.

65. If $A = SDS^{-1}$, where $S = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, find A^p for any positive integer p.

(a) $\begin{bmatrix} 2^p - 3^p & 3^p \\ 2^p & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2^p & 2^p - 3^p \\ 0 & 3^p \end{bmatrix}$ (c) $\begin{bmatrix} 2^p & 0 \\ 2^p - 3^p & 3^p \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2^p \\ 3^p & 2^p - 3^p \end{bmatrix}$

(e) $\begin{bmatrix} 3^p & 0 \\ 2^p - 3^p & 2^p \end{bmatrix}$ (f) $\begin{bmatrix} 3^p & 0 \\ 3^p - 2^p & 2^p \end{bmatrix}$.

66. A certain 3×3 matrix A has eigenvalue 5 with algebraic multiplicity 3. Which one of the following matrices is similar to A?

(a) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (e) $\begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$ (f) not enough information.

67. A 3×3 matrix A has eigenspaces $E_1 = \mathbb{R} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \mathbb{R} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ and $E_2 = \mathbb{R} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Which of the following matrices S does NOT make $S^{-1}AS$ diagonal?

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 2 & 1 & 1 \\ 4 & 1 & 1 \\ -5 & -1 & 0 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ (h) $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.

68. A certain 3×3 matrix A has only the eigenspaces $E_1 = \mathbb{R} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $E_2 = \mathbb{R} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Find an invertible matrix S such that $S^{-1}AS$ is diagonal.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ (f) $\begin{bmatrix} 2 & 1 & 0 \\ 4 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

(g) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (h) $\begin{bmatrix} 0 & 0 & 1 \\ 2 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ (i) no such matrix.

69. A symmetric 3×3 matrix A has eigenspaces $E_4 = \mathbb{R} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $E_1 = \mathbb{R} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \mathbb{R} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Find an <u>orthogonal</u> matrix S such that $S^{-1}AS$ is diagonal.

(a) $\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (e) not enough information.

I plan to post the answers to the multiple-choice problems herein at my **homework** website (http://www.uta.edu/math/vancliff/T/F15/hwk3330.html) by 2:00 pm Sun Dec 13, so work the problems first and then compare with the posted answers.

You might find it beneficial to pick 14 of these questions and answer them in a simulated "test environment"; i.e., time yourself for 30 minutes and do not use your notes or book etc. This will allow your brain/mind practice what it will be like to answer Math 3330 questions in a test environment.

Remember: most of the test will be based on the homework questions, so finish those questions and look over your solutions to them before the test. Reread Page 1 above to see what to bring to the test.

Recall that I have extra office hours starting on Saturday Dec 12 – check page 1 above.

I would like to take this opportunity to remind y'all to complete the student feedback survey for all your classes (& definitely for this class!) by 11 pm Wed Dec 9; see your mymav email for the link. Thank you!