Test 1 will be in PKH 102 in class time, Thurs Sept 19

- See above for date, time and location of Test 1. It will last 20 minutes and is worth 12% of your course grade.
- The material to be covered on Test 1 is the same as that covered on the homework through §2.2 inclusive. (Homework is listed at my website: www.uta.edu/math/vancliff/T/F13 .)
- My office hours are: Tues & Thurs 3:30-4:10 pm.
- This test will be entirely multiple choice, but you do NOT need to bring a scantron form. There will be several choices of answer per multiple-choice question and, for each, only one answer will be the correct one. You should do rough work on the test or on paper provided by me. No calculator is allowed. No notes or cards are allowed.

  BRING YOUR MYMAV ID CARD WITH YOU.
- When I write a test, I look over the lecture notes and homework which have already been assigned, and use them to model about 85% of the test problems (and most of them are fair game). You should expect 8-10 questions in total.
- A good way to review is to go over the homework problems you have not already done & make sure you understand all the homework well by 48 hours prior to the test. You could also go over the quiz; the quiz and its solutions are posted online at my website in a single pdf file (use password "math3330" to open the file); see http://www.uta.edu/math/vancliff/T/F13/3330q.pdf . In addition, this information sheet provides some practice problems that are provided to help you study, if you have finished all the homework questions. These practice questions do NOT form a model for the test. These questions are intended only to help you identify any gaps in your understanding. In the last 24 hours before the test, reread ALL the homework problems, skim through the lecture notes, & go over these practice questions again.
- Try to keep your eyes on your own work during the test.
- All nontest items will have to be put by the wall or by my desk prior to the start of the test; this includes cell phones, which should be switched off and in with your belongings by the wall.
- Any student who leaves the room during the test will not be allowed to continue the test. If you finish the test early, & wish to leave early, then please do so as quietly as possible. Please turn in your test and any additional paper (used or unused) when you leave.
- It is your responsibility to be on time.
- I hope to return the graded test to you on Tues Sept 24. We will have a lecture as usual on Tues Sept 24.

The following questions are for your practice for the test to see how I might phrase some questions (especially multiple choice); they will not be graded and do NOT form a model for the test. Finish the homework assignments first, before working them.

PRACTICE QUESTIONS

1. Which of the following systems of equations is useful for determining the function(s) $f(t) = a + bt + ct^2$ whose graph passes through the points $(0,\ 2)$, $(1,\ 8)$, $(-1,\ 18)$?

(a) $\begin{aligned} a &= -2 \\ a + 4b &= -8 \\ a - b + c &= -18 \end{aligned}$ (b) $\begin{aligned} b &= 2 \\ a + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (c) $\begin{aligned} a &= 2 \\ a + 4b &= 8 \\ a - b + c &= 18 \end{aligned}$

(d) $\begin{aligned} a &= -2 \\ a + b + c &= -8 \\ a - b + c &= -18 \end{aligned}$ (e) $\begin{aligned} a &= 2 \\ a + b + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (f) $\begin{aligned} a + b &= 2 \\ a + c &= 8 \\ b - c &= 18. \end{aligned}$

2. Which of the following systems of equations is useful for determining the function(s) $f(t) = a\cos(2t) + b\sin(2t)$ that satisfy the equation $\dfrac{d^2f}{dt^2} + 2\dfrac{df}{dt} + 3f(t) = 5\cos(2t)$ for all $t \in \mathbb{R}$?

(a) $\begin{aligned} 4a - b &= 0 \\ a + 4b &= -5 \end{aligned}$ (b) $\begin{aligned} 4a + b &= 0 \\ a - 4b &= -5 \end{aligned}$ (c) $\begin{aligned} a + 4b &= 0 \\ a + 4b &= -5 \end{aligned}$ (d) $\begin{aligned} a + 4b &= 0 \\ a + b &= -5 \end{aligned}$

(e) $\begin{aligned} a + b &= 0 \\ a + 2b &= -5 \end{aligned}$ (f) $\begin{aligned} a - b &= 0 \\ a - 2b &= -5. \end{aligned}$

3. Write down the coefficient matrix and the augmented matrix (stating which is which) for the system
$$\begin{aligned} x_1 - x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 &= -1 \\ x_2 - 2x_3 &= 1. \end{aligned}$$

4. The system of linear equations
$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 1 \\ 2x_2 + x_3 &= 1 \\ x_2 + x_3 &= 1 \end{aligned}$$
has

(a) no solution (b) exactly 1 solution (c) infinitely many solutions (d) exactly 2 solutions.

5. Find the rank of $\begin{bmatrix} 0 & 1 & 5 \\ 1 & 0 & -7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) 0 (b) 1 (c) 2 (d) 3
(e) 4 (f) 4.5 (g) 5 (h) 6.

6. Which one of the following explains why $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ is <u>not</u> in reduced row-echelon form?

   (a) Column 2 contains a nonzero entry.
   (b) Column 3 contains a pivot and another nonzero entry.
   (c) Column 4 contains a pivot and another nonzero entry.
   (d) Row 1 contains a pivot and another nonzero entry.
   (e) Row 3 contains a pivot and another nonzero entry.
   (f) The number of rows does not equal the number of columns.
   (g) There is not a row of zeros in the matrix.
   (h) Trick question: the matrix is in reduced row-echelon form.

7. Find the reduced row-echelon form of $\begin{bmatrix} 2 & 4 & 6 & -8 \\ 3 & 6 & 7 & 10 \end{bmatrix}$.

   (a) $\begin{bmatrix} 1 & 2 & 3 & -4 \\ 1 & 2 & \frac{7}{3} & \frac{10}{3} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 22 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 0 & 37 \\ 0 & 0 & 1 & -11 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 0 & 29 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 2 & 3 & -4 \\ 0 & 0 & 1 & -11 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & 0 & -11 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ (h) $\begin{bmatrix} 1 & 2 & 0 & 29 \\ 0 & 0 & 1 & -11 \end{bmatrix}$.

8. If $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, then which of the following is a nontrivial (i.e., nonzero) solution of the equation $Ax = 0$? (a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (g) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (h) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

9. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the set of solutions to $Ax = 0$ geometrically?

   (a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) a plane in $\mathbb{R}^4$ (d) a plane in $\mathbb{R}^3$
   (e) a line in $\mathbb{R}^3$ (f) $\mathbb{R}^3$ (g) $\mathbb{R}^4$ (h) 3 points in $\mathbb{R}^3$.

10. If $\begin{bmatrix} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 5 & k \end{bmatrix}$ is the augmented matrix for a system of linear equations, then for which value(s) of $k \in \mathbb{R}$ is the system consistent?

    (a) all $k \in \mathbb{R}$ except $k = 1$ (b) $k = 1$ (c) $k = 2$ (d) $k = 4$
    (e) all $k \in \mathbb{R}$ except $k = 2$ (f) all $k \in \mathbb{R}$ except $k = 4$.

11. Justify or disprove: if a given vector b is a linear combination of the columns of a matrix A, then the equation $Ax = b$ is consistent.

12. The product $\begin{bmatrix} 1 & 3 & 5 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$ equals

(a) $3\begin{bmatrix}1\\0\end{bmatrix}+7\begin{bmatrix}3\\-1\end{bmatrix}+4\begin{bmatrix}5\\2\end{bmatrix}$ (b) $-3\begin{bmatrix}1\\0\end{bmatrix}-7\begin{bmatrix}3\\-1\end{bmatrix}-4\begin{bmatrix}5\\2\end{bmatrix}$ (c) $4\begin{bmatrix}1\\0\end{bmatrix}+7\begin{bmatrix}3\\-1\end{bmatrix}+3\begin{bmatrix}5\\2\end{bmatrix}$

(d) $-4\begin{bmatrix}1\\0\end{bmatrix}-7\begin{bmatrix}3\\-1\end{bmatrix}-3\begin{bmatrix}5\\2\end{bmatrix}$ (e) $\begin{bmatrix}5\\4\end{bmatrix}+\begin{bmatrix}10\\6\end{bmatrix}+\begin{bmatrix}8\\5\end{bmatrix}$ (f) trick question: this product is not defined.

13. If $\begin{bmatrix}1&0&3&0&3\\0&1&2&0&4\\0&0&0&1&5\end{bmatrix}$ is the augmented matrix for a system of linear equations, then the solution set of the system in vector form is:

(a) $x=\begin{bmatrix}3\\4\\5\\0\end{bmatrix}+x_3\begin{bmatrix}-3\\-2\\1\\0\end{bmatrix}$ (b) $x=\begin{bmatrix}3\\4\\0\\5\end{bmatrix}+x_3\begin{bmatrix}-3\\-2\\1\\0\end{bmatrix}$ (c) $x=\begin{bmatrix}3\\4\\5\\0\end{bmatrix}+x_3\begin{bmatrix}-3\\-2\\0\\1\end{bmatrix}$

(d) there are no solutions (e) not enough information given to answer question.

14. A system of linear equations has augmented matrix

$$\left[\begin{array}{ccc:c}1&1&1&2\\0&1&k-1&2\\0&0&(1-k)(3k-2)&6(1-k)\end{array}\right],$$

where $k\in\mathbb{R}$. Find all values of k such that the system does NOT have a unique solution.

(a) all $k\in[\frac{2}{3},\ 1]$ (b) $\pm\frac{2}{3}$, ±1 only (c) $\frac{2}{3}$, 1, $\frac{4}{3}$ only (d) $\frac{4}{3}$ only (e) $\frac{2}{3}$ only

(f) 1 only (g) $\frac{2}{3}$, 1 only (h) trick question: there is a unique solution for all $k\in\mathbb{R}$.

15. Let A denote an $m\times n$ matrix and let $x\in\mathbb{R}^n$. Which of the following statements is always true?

(a) The system $Ax=0$ is always consistent.
(b) The system $Ax=0$ is never consistent.
(c) The system $Ax=0$ always has nonzero solutions.
(d) If A is a 3×2 matrix, then A must have a column without a pivot.

16. Given that the reduced row-echelon form of $\begin{bmatrix}1&2&12\\0&1&5\\1&1&k\end{bmatrix}$ is $\begin{bmatrix}1&0&2\\0&1&5\\0&0&k-7\end{bmatrix}$,

for which value(s) of k is $\begin{bmatrix}12\\5\\k\end{bmatrix}$ a linear combination of $\begin{bmatrix}1\\0\\1\end{bmatrix}$ and $\begin{bmatrix}2\\1\\1\end{bmatrix}$?

(a) all $k\in\mathbb{R}$ except 7 (b) $k=0$ only (c) $k=7$ only (d) $k=0$ & $k=7$ only

(e) all $k\in\mathbb{R}$ except 0 & 7 (f) all $k\in\mathbb{R}$ (g) there is no such k (h) not enough information given.

17. Define $T:\mathbb{R}^2\to\mathbb{R}^2$ by $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{cases}\begin{bmatrix}x_1\\\frac{x_2^2}{x_1}\end{bmatrix} & \text{if } x_1\neq0\\ \begin{bmatrix}0\\x_2\end{bmatrix} & \text{if } x_1=0.\end{cases}$

Show that there exist $u, v \in \mathbb{R}^2$ such that $T(u+v) \neq T(u)+T(v)$. (Hint: consider $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}1\\0\end{bmatrix}$.)

18. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \left(\begin{bmatrix}2\\5\end{bmatrix}\right)$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \left(\begin{bmatrix}-1\\6\end{bmatrix}\right)$, find $T\left(\begin{bmatrix}5\\-3\end{bmatrix}\right)$.

(a) $\begin{bmatrix}13\\7\end{bmatrix}$ (b) $\begin{bmatrix}-7\\43\end{bmatrix}$ (c) $\begin{bmatrix}-13\\-7\end{bmatrix}$ (d) $\begin{bmatrix}10\\15\end{bmatrix}$ (e) $\begin{bmatrix}0\\0\end{bmatrix}$ (f) $\begin{bmatrix}2\\23\end{bmatrix}$.

19. Let $u,\ v \in \mathbb{R}^3$. If $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation such that $T(u) = \begin{bmatrix}1\\2\end{bmatrix}$ and $T(v) = \begin{bmatrix}-3\\2\end{bmatrix}$, find $T(2u-3v)$.

(a) $\begin{bmatrix}-2\\4\end{bmatrix}$ (b) $\begin{bmatrix}4\\2\end{bmatrix}$ (c) $\begin{bmatrix}11\\-2\end{bmatrix}$ (d) $\begin{bmatrix}1\\1\end{bmatrix}$ (e) $\begin{bmatrix}3\\0\end{bmatrix}$ (f) not enough information is given.

20. Find the matrix of the function $T : \mathbb{R}^2 \to \mathbb{R}$, where $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = x_2^2$.

(a) $\begin{bmatrix}x_1 & x_2^2\end{bmatrix}$ (b) $\begin{bmatrix}0 & 1\end{bmatrix}$ (c) $\begin{bmatrix}0 & 0\\0 & x_2^2\end{bmatrix}$ (d) $\begin{bmatrix}0 & x_2\end{bmatrix}$ (e) $\begin{bmatrix}x_2^2\end{bmatrix}$ (f) no such matrix.

21. Suppose a line ℓ in $\mathbb{R}^2$ contains the vector $u = \begin{bmatrix}-0.6\\0.8\end{bmatrix}$ of length 1. Find the matrix of the (orthogonal) projection map onto ℓ.

(a) $\begin{bmatrix}0.64 & -0.48\\-0.48 & 0.36\end{bmatrix}$ (b) $\begin{bmatrix}0.48 & -0.64\\-0.64 & 0.36\end{bmatrix}$ (c) $\begin{bmatrix}0.36 & -0.48\\-0.48 & 0.64\end{bmatrix}$

(d) $\begin{bmatrix}0.48 & -0.36\\-0.36 & 0.64\end{bmatrix}$ (e) $\begin{bmatrix}0.64 & -0.36\\-0.36 & 0.48\end{bmatrix}$ (f) no such matrix.

22. Find the shear matrix that transforms $\begin{bmatrix}3\\1\end{bmatrix}$ into $\begin{bmatrix}3\\16\end{bmatrix}$.

(a) $\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$ (b) $\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$ (c) $\begin{bmatrix}5 & 0\\1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 5\\0 & 1\end{bmatrix}$ (e) $\begin{bmatrix}1 & 1\\0 & 5\end{bmatrix}$ (f) no such shear.

23. If $A = \begin{bmatrix}k & 2\\-3 & 4\end{bmatrix}$, then for which value(s) of k is A not invertible?

(a) 2 (b) $\frac{3}{2}$ (c) -2 (d) $-\frac{3}{2}$ (e) 0 (f) 0 & 2.

I plan to post the answers to the multiple-choice problems herein at my website roughly 24 hours before Thursday's class, so work the problems first and then compare with the posted answers.

Remember: most of the test will be based on the homework questions, so finish those questions and look over your solutions to them before the test. Reread Page 1 above to see what to bring to the test.