Examples from end of §7.1 not covered in class

Eg A from c-r example. $Av_1 = 1.1v_1$ $Av_2 = 0.9v_2$ $\{v_1, v_2\}$ = ebasis.

λ_1 λ_2

$B = S^{-1}AS$ $B = \begin{bmatrix} 1.1 & 0 \\ 0 & 0.9 \end{bmatrix}$, $S = \begin{bmatrix} v_1 & v_2 \end{bmatrix} = \begin{bmatrix} 100 & 200 \\ 300 & 100 \end{bmatrix}$

Eg A from c-r example. $v_1' = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \frac{1}{100} v_1$, $v_2' = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{100} v_2$, $\{v_1', v_2'\}$ = ebasis.

$B = S^{-1}AS = \begin{bmatrix} 1.1 & 0 \\ 0 & 0.9 \end{bmatrix}$, $S = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. So S not unique.

Eg A from c-r example. Switch order of v_1', v_2' $\{v_2', v_1'\}$ = ebasis.

$B = S^{-1}AS = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix}$, $S = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

Eg $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ projection onto x-axis. $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$ $\forall\, x, y \in \mathbb{R}$

evecs: $\begin{bmatrix} x \\ 0 \end{bmatrix}$ w/ $\lambda = 1$, $\forall\, x \in \mathbb{R}$, $x \neq 0$. $\begin{bmatrix} 0 \\ y \end{bmatrix}$ w/ $\lambda = 0$. $\forall\, y \in \mathbb{R}$, $y \neq 0$.

Eg $T: \mathbb{R}^2 \to \mathbb{R}^2$ rotation cc by 90°.

$T(v) \perp v$ $\forall\, v \in \mathbb{R}^2$ $\Rightarrow$ $T(v) \not\parallel v$, $\forall\, v \in \mathbb{R}^2$ $\Rightarrow$ no evecs of T in $\mathbb{R}^2$.
no evals of T in $\mathbb{R}$.

Eg $A = I_n$. $Av = v = 1v$ $\forall\, v \in \mathbb{R}^n$.

$\Rightarrow$ all nonzero $v \in \mathbb{R}^n$ are evecs of A w/ have eval $= 1$.

$\Rightarrow$ all bases of $\mathbb{R}^n$ are ebases for A. $S^{-1} I_n S = S^{-1} S = I_n$.