FINAL EXAM will be in PKH 319 at 5:30-8:00 pm on Tues Dec 13

1. The topics covered on the Final Exam will be the topics covered in the homework. The homework is online; see http://www.uta.edu/math/vancliff/T/F11 .

2. I will have an office hour on Mon Dec 12 at 3:30-5:30 pm in PKH 462.

3. You may expect at least half of the test to be modelled on the homework questions. You should expect the test to be roughly double the length of the midterm. Conceivably, questions occurring on earlier tests could be on the test, so looking over the earlier tests is always wise.

4. You should know the definitions of the concepts (and examples thereof) that arise frequently in class; the theorems etc that are used frequently or which have a name; how to apply the theorems to examples and at least one method for doing each homework question. Skimming over short proofs (at most 6 or so lines) from lecture could also be helpful. As stated above, looking over earlier tests is also a wise thing to do.

5. Any student who leaves the room during the test will not be allowed to continue the test. If you finish the test early, then your fellow students would prefer that you NOT distract their concentration by leaving the room early. Instead, stay in your seat, and look over your work and check your answers.

6. Cellular phones should be switched off during the test and it is your responsibility to be on time. I will probably ask y'all to spread yourselves around the room before starting the test.

7. Recall from the first-day handout that any student not obtaining a positive score on the Final Exam will not pass this class.

8. If you wish to know your grade early, turn in to me a STAMPED ADDRESSED envelope with the following listed on a piece of paper inside:

   (a) MATH 5333

   (b) Your full name written clearly

   (c) The last 4 digits of your ID number

   (d) Grade on Final Exam =

   (e) Grade in course =

   I will <u>not</u> give out any grades by e-mail.

In the following, $\mathbb{F}$ denotes any field.

In order to solve systems of linear equations with coefficients in $\mathbb{F}$, we consider $\mathbb{F}^n$ and $m \times n$ matrices with entries in $\mathbb{F}$. This leads us to the notion of vector space over $\mathbb{F}$ and linear maps from one vector space to another. To eliminate redundancy in a vector space and to find "simple" ways to describe vector spaces, we consider linear (in)dependence, spanning sets and bases. (To solve differential equations, one sometimes needs to consider infinite-dimensional vector spaces, which are studied in detail in functional analysis.)

If the vector space is finite-dimensional, then a linear map from it to itself is represented by a square matrix. In this case, we use the notion of eigenvector and eigenvalue to understand the linear map better. The notions of eigenvector and eigenvalue lead us to eigenspaces and invariant subspaces. We will return to these ideas in the last few weeks of the semester.

Recently we have been considering special types of vector spaces, namely those that have an inner product on them. The inner product allows us to define the notion of norm (e.g., length) & orthogonal vectors. This brings us to the notions of orthonormal bases & orthogonal complement of a subset or of a subspace. This, in turn, allows us to find a vector in a subspace that is as "close as possible" to a given generic vector in our inner-product space. This latter type of situation is useful in statistics, engineering, geology and some other applied fields.

In the last couple of weeks we have concentrated our attention on linear operators on inner-product spaces. In particular, we looked at the adjoint of a linear operator, and considered self-adjoint operators and normal operators. These last two types of operators have important theorems about them called the real spectral theorem and the complex spectral theorem respectively; both theorems address when the operator has an orthonormal eigenbasis (and so the operator can be given by a diagonal matrix).

The rest of the semester was devoted to what happens if a linear operator does not have an eigenbasis. In this case, the operator cannot be given by a diagonal matrix, so we found the next best type of matrix; we called it the Jordan normal (canonical) form of the operator. In this case, the basis consists of eigenvectors and generalized eigenvectors. In this case, starting with a matrix A and changing to the Jordan normal form J uses the formula $J = P^{-1}AP$, where the columns of the matrix P are given by the new basis that consists of eigenvectors and generalized eigenvectors of A.

We also reconsidered the real spectral theorem again and used the Gram-Schmidt orthogonalization procedure to find a change-of-basis matrix P for a real symmetric matrix A such that the columns of P are orthonormal eigenvectors of A. In this case, the formula above becomes $J = P^{-1}AP$, where P is orthogonal, and hence $P^T = P$ so that $J = P^TAP$.