TEST 3 is in PKH 103 in class time, Thurs Dec 1

- See above for date, time and location of TEST 3. It will last 20 minutes.
- The material covered on Test 3 is the same as that covered on the homework through §7.2 inclusive. (Homework is listed at my website: www.uta.edu/math/vancliff/T/F11 .)
- My office hours are: Tues & Thurs 3:30-4:20 pm.
- This test will be, in part, multiple choice, but you do NOT need to bring a scantron form. There will be several choices of answer per multiple-choice question and, for each, only one answer will be the correct one. You should do rough work on the test or on paper provided by me. No calculator is allowed. No notes or cards are allowed.

  BRING YOUR MYMAV ID CARD WITH YOU.
- When I write a test, I look over the lecture notes and homework which have already been assigned, and use them to model about 85% of the test problems (and most of them are fair game). You should expect approximately 10 questions in total.
- A good way to review is to go over the homework problems you have not already done & make sure you understand all the homework well by 48 hours prior to the test. You should also look over the past tests/midterm and understand those fully. In addition, this information sheet provides some practice problems that are provided to help you study if you have finished all the homework questions. These practice questions do NOT form a model for the test. These questions are intended only to help you identify any gaps in your understanding. In the last 24 hours before the test, reread ALL the homework problems, skim through the lecture notes and past tests/midterm, & go over these practice questions again.
- Try to keep your eyes on your own work during the test.
- All nontest items will have to be put by the wall or on my desk prior to the start of the test; this includes cell phones, which should be switched off.
- Any student who leaves the room during the test will not be allowed to continue the test. If you finish the test early, & wish to leave early, then please do so as quietly as possible. Please turn in your test and any additional paper (used or unused) when you leave.
- It is your responsibility to be on time.
- I hope to return the graded test to you on Tues Dec 6 with a solution sheet. We will have a lecture as usual on Tues Dec 6.

The following questions are for your practice for the test to see how I might phrase some questions (especially multiple choice); they will not be graded and do NOT form a model for the test. Finish the homework assignments first, before working them.

PRACTICE QUESTIONS

1. Which of the following systems of equations is useful for determining the function(s) $f(t) = a + bt + ct^2$ whose graph passes through the points $(0,\ 2),\ (1,\ 8),\ (-1,\ 18)$?

(a) $\begin{aligned} a &= -2 \\ a + 4b &= -8 \\ a - b + c &= -18 \end{aligned}$ (b) $\begin{aligned} a &= 2 \\ a + b + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (c) $\begin{aligned} a &= 2 \\ a + 4b &= 8 \\ a - b + c &= 18 \end{aligned}$

(d) $\begin{aligned} a &= -2 \\ a + b + c &= -8 \\ a - b + c &= -18 \end{aligned}$ (e) $\begin{aligned} b &= 2 \\ a + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (f) $\begin{aligned} a + b &= 2 \\ a + c &= 8 \\ b - c &= 18. \end{aligned}$

2. Which of the following systems of equations is useful for determining the function(s) $f(t) = a\cos(2t) + b\sin(2t)$ that satisfy the equation $\dfrac{d^2f}{dt^2} + 2\dfrac{df}{dt} + 3f(t) = 5\cos(2t)$ for all $t \in \mathbb{R}$?

(a) $\begin{aligned} 4a - b &= 0 \\ a + 4b &= -5 \end{aligned}$ (b) $\begin{aligned} 4a + b &= 0 \\ a - 4b &= -5 \end{aligned}$ (c) $\begin{aligned} a + 4b &= 0 \\ a + 4b &= -5 \end{aligned}$ (d) $\begin{aligned} a + 4b &= 0 \\ a + b &= -5 \end{aligned}$

(e) $\begin{aligned} a + b &= 0 \\ a + 2b &= -5 \end{aligned}$ (f) $\begin{aligned} a - b &= 0 \\ a - 2b &= -5. \end{aligned}$

3. Write down the coefficient matrix and the augmented matrix (stating which is which) for the system
$$\begin{aligned} x_1 - x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 &= -1 \\ x_2 - 2x_3 &= 1. \end{aligned}$$

4. Which of the following matrices is not in row-echelon form?

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 4 & 5 & 6 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 8 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 4 & 8 & 8 \\ 0 & 0 & 9 & 10 \\ 0 & 2 & 3 & 4 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

5. If $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, then which of the following is a nontrivial solution of the equation $Ax = 0$?

(a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (g) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (h) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

6. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the set of solutions to $Ax = 0$ geometrically?

   (a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) a line in $\mathbb{R}^3$ (d) a plane in $\mathbb{R}^3$
   (e) a plane in $\mathbb{R}^4$ (f) $\mathbb{R}^3$ (g) $\mathbb{R}^4$ (h) 3 points in $\mathbb{R}^3$.

7. The system of linear equations $\begin{aligned} x_1 + 4x_2 + 3x_3 &= 1 \\ 2x_2 + x_3 &= 1 \\ x_2 + x_3 &= 1 \end{aligned}$ has

   (a) no solution (b) exactly 1 solution (c) infinitely many solutions (d) exactly 2 solutions.

8. If $\begin{bmatrix} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 5 & k \end{bmatrix}$ is the augmented matrix for a system of linear equations, then for which value(s) of $k \in \mathbb{R}$ is the system consistent?

   (a) $k = 1$ (b) $k = 2$ (c) $k = 4$ (d) all $k \in \mathbb{R}$ except $k = 1$
   (e) all $k \in \mathbb{R}$ except $k = 2$ (f) all $k \in \mathbb{R}$ except $k = 4$.

9. Justify or disprove: if a given vector b is a linear combination of the columns of a matrix A, then the equation $Ax = b$ is consistent.

10. If $\begin{bmatrix} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$ is the augmented matrix for a system of linear equations, then the solution set of the system in vector form is:

    (a) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (b) $x = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (c) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

    (d) there are no solutions (e) not enough information given to answer question.

11. Which of the following statements is always true?

    (a) A homogeneous system of linear equations is always consistent.
    (b) A homogeneous system of linear equations is never consistent.
    (c) A homogeneous system of linear equations always has nontrivial solutions.
    (d) If A is a 3×2 matrix, then A must have a column without a pivot.

12. Compute $\begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

    (a) $\begin{bmatrix} 2 & 2 & 7 \\ -2 & 4 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 6 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & -2 \\ 2 & 4 \\ 7 & 10 \end{bmatrix}$

    (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} -1 & 2 & -3 \\ -1 & 6 & -5 \end{bmatrix}$ (f) $\begin{bmatrix} -1 & -1 \\ 2 & 6 \\ -3 & -5 \end{bmatrix}$ (g) cannot be multiplied.

13. Noting that the vector $v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is a particular solution to the system $\begin{aligned} 2x_1 - x_2 + x_3 &= 0 \\ 3x_1 + 2x_2 - x_3 + x_4 &= 1 \\ 5x_1 + 8x_2 - 5x_3 + 3x_4 &= 3, \end{aligned}$

find *all* solutions to the system.

(a) $v + s\begin{bmatrix} 1 \\ 0 \\ -2 \\ -5 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ (b) $v + s\begin{bmatrix} 1 \\ 0 \\ 2 \\ -5 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ (c) $v + s\begin{bmatrix} 1 \\ 0 \\ -2 \\ -5 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(d) none of the above.

14. Given that the reduced row-echelon form of $\begin{bmatrix} 1 & 2 & 12 \\ 0 & 1 & 5 \\ 1 & 1 & k \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & k-7 \end{bmatrix}$,

for which value(s) of k is $\begin{bmatrix} 12 \\ 5 \\ k \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$?

(a) all $k \in \mathbb{R}$ except 7 (b) $k = 0$ only (c) $k = 7$ only (d) $k = 0$ & $k = 7$ only

(e) all $k \in \mathbb{R}$ except 0 & 7 (f) all $k \in \mathbb{R}$ (g) there is no such k (h) not enough information given.

15. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{cases} \begin{bmatrix} x_1 \\ \frac{x_2^2}{x_1} \end{bmatrix} & \text{if } x_1 \neq 0 \\ \begin{bmatrix} 0 \\ x_2 \end{bmatrix} & \text{if } x_1 = 0. \end{cases}$

Show that there exist $u, v \in \mathbb{R}^2$ such that $T(u+v) \neq T(u) + T(v)$. (Hint: consider $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.)

16. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right)$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \left(\begin{bmatrix} -1 \\ 6 \end{bmatrix}\right)$, find $T\left(\begin{bmatrix} 5 \\ -3 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} 13 \\ 7 \end{bmatrix}$ (b) $\begin{bmatrix} -7 \\ 43 \end{bmatrix}$ (c) $\begin{bmatrix} -13 \\ -7 \end{bmatrix}$ (d) $\begin{bmatrix} 10 \\ 15 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 2 \\ 23 \end{bmatrix}$.

17. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that $T(u) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(v) = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, find $T(2u - 3v)$.

(a) $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ (f) not enough information is given.

18. Find the matrix of the function $T : \mathbb{R}^2 \to \mathbb{R}$, where $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_2^2$.

(a) $\begin{bmatrix} x_1 & x_2^2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & x_2^2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & x_2 \end{bmatrix}$ (e) $\begin{bmatrix} x_2^2 \end{bmatrix}$ (f) no such matrix.

19. Suppose a line ℓ in $\mathbb{R}^2$ contains the vector $u = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$ of length 1. Find the matrix of the (orthogonal) projection map onto ℓ.

(a) $\begin{bmatrix} 0.64 & -0.48 \\ -0.48 & 0.36 \end{bmatrix}$ (b) $\begin{bmatrix} 0.48 & -0.64 \\ -0.64 & 0.36 \end{bmatrix}$ (c) $\begin{bmatrix} 0.36 & -0.48 \\ -0.48 & 0.64 \end{bmatrix}$

(d) $\begin{bmatrix} 0.48 & -0.36 \\ -0.36 & 0.64 \end{bmatrix}$ (e) $\begin{bmatrix} 0.64 & -0.36 \\ -0.36 & 0.48 \end{bmatrix}$ (f) no such matrix.

20. Find the shear matrix that transforms $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 16 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 \\ 0 & 5 \end{bmatrix}$ (f) no such shear.

21. If $A = \begin{bmatrix} k & 2 \\ -3 & 4 \end{bmatrix}$, then for which value(s) of k is A not invertible?

(a) 2 (b) $\frac{3}{2}$ (c) -2 (d) $-\frac{3}{2}$ (e) 0 (f) 0 & 2.

22. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 2 & 1 & 4 \end{bmatrix}$, then which of the following is the second row of A^{-1}?

(a) 1 1 1 (b) 0 0 1 (c) 4 -2 -3 (d) 1 2 -3 (e) 2 3 4 (f) 4 5 -6
(g) 0 -2 1 (h) 0 2 1 (i) A^{-1} does not exist.

23. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 7 \\ 10 \\ 9 \end{bmatrix}$. Describe $\text{span}(v_1, v_2, v_3) = \mathbb{R}v_1 + \mathbb{R}v_2 + \mathbb{R}v_3$.

(a) a point (b) a line (c) a plane (d) $\mathbb{R}^3$ (e) exactly 3 points
(f) exactly 3 vectors emerging from the origin in $\mathbb{R}^3$.

24. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the kernel of A geometrically?

(a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) 3 points in $\mathbb{R}^3$
(d) a line in $\mathbb{R}^3$ (e) a plane in $\mathbb{R}^3$ (f) a plane in $\mathbb{R}^9$ (g) $\mathbb{R}^3$ (h) $\mathbb{R}^4$.

25. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the image of A geometrically?

(a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) 3 points in $\mathbb{R}^3$

(d) a line in $\mathbb{R}^3$ (e) a plane in $\mathbb{R}^3$ (f) a plane in $\mathbb{R}^9$ (g) $\mathbb{R}^3$ (h) $\mathbb{R}^4$.

26. The set $U = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ 5 \end{bmatrix} : x_1, x_2 \in \mathbb{R} \right\}$ is not a subspace of $\mathbb{R}^3$ because

(a) U is not a subset of $\mathbb{R}^3$ (b) U is empty (c) U contains $\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$ (d) U contains the origin

(e) U does not contain the origin (f) trick question: U is a subspace of $\mathbb{R}^3$.

27. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, $v_3 = \begin{bmatrix} 9 \\ 2 \\ 33 \end{bmatrix}$. Given that $\begin{bmatrix} 5 \\ -7 \\ 1 \end{bmatrix}$ is in the kernel of the matrix $A = \begin{bmatrix} 1 & 2 & 9 \\ 1 & 1 & 2 \\ -1 & 4 & 33 \end{bmatrix}$, find a nontrivial relation among v_1, v_2 and v_3.

Show some work; write clearly.

28. Which of the following sets of vectors is linearly independent?

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ (f) none of these.

29. Which of the sets in the previous question is a basis for $\mathbb{R}^3$? (a) (b) (c) (d) (e) (f) none.

30. Define (i) subspace of $\mathbb{R}^n$ (ii) span of m vectors in $\mathbb{R}^n$ (iii) kernel of a linear transformation (iv) image of a linear transformation (v) basis of a subspace (vi) dimension of a subspace (vii) the coordinate vector of a vector with respect to a basis $\mathcal{B}$.

31. The set $W = \left\{ \begin{bmatrix} d \\ c \\ b \\ a \end{bmatrix} : a, b, c, d \in \mathbb{R} \text{ and } a + d = b + c \right\}$ is a subspace of $\mathbb{R}^4$. Find the dimension of W.

(a) 7 (b) 6 (c) 5 (d) 4 (e) 3 (f) 2 (g) 1 (h) 0.

32. You are given a matrix A and its reduced row-echelon form:

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 & 0 & -1 & 4 \\ 1 & 1 & 3 & 0 & 0 & 2 & 3 \\ 1 & 1 & 3 & 1 & 0 & 1 & 7 \\ 1 & 1 & 3 & 1 & 1 & 4 & 6 \\ 1 & 1 & 3 & 1 & 1 & 4 & 6 \\ 1 & 1 & 3 & 1 & 1 & 4 & 6 \end{bmatrix} \qquad \text{rref}(A) = \begin{bmatrix} 1 & 0 & 3 & 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The matrix A represents a linear transformation T where $T(x) = Ax$ and....

(a) $T : \mathbb{R}^{13} \to \mathbb{R}^6$ (b) $T : \mathbb{R}^6 \to \mathbb{R}^{13}$ (c) $T : \mathbb{R}^7 \to \mathbb{R}^6$ (d) $T : \mathbb{R}^{42} \to \mathbb{R}^{42}$

(e) $T : \mathbb{R}^6 \to \mathbb{R}^7$ (f) $T : \mathbb{R}^7 \to \mathbb{R}^{13}$ (g) $T : \mathbb{R}^{13} \to \mathbb{R}^7$ (h) not enough information.

33. Find the dimension of the image of the matrix A given in Question 32.

(a) 6 (b) 5 (c) 4 (d) 3 (e) 2 (f) 1 (g) 0 (h) not enough information.

34. Find a basis for the image of the matrix A given in Question 32.

(a) Rows 1, 2, 4 of rref(A) (b) rows 1-4 of rref(A)

(c) rows 1, 2, 4, 5 of A (d) all the rows of A

(e) columns 1, 2, 4, 5 of rref(A) (f) all the columns of rref(A)

(g) columns 1, 2, 4, 5 of A (h) all the columns of A.

35. Using the matrix A from Question 32 and a vector x, how many free variables has the homogeneous system $Ax = 0$?

(a) 7 (b) 6 (c) 5 (d) 4 (e) 3 (f) 2 (g) 1 (h) 0.

36. Find the dimension of the kernel of the matrix A given in Question 32.

(a) 7 (b) 6 (c) 5 (d) 4 (e) 3 (f) 2 (g) 1 (h) 0.

37. Find a basis for the kernel of the matrix A given in Question 32; show all work.

38. Let $A = \begin{bmatrix} -3 & 10 \\ -3 & 8 \end{bmatrix}$, $v_1 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Using $\mathcal{B} = \{v_1, v_2\}$ as a basis for $\mathbb{R}^2$, find $[Av_2]_{\mathcal{B}}$.

(a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (f) not enough information.

39. Let $T(x) = Ax$ for all $x \in \mathbb{R}^2$, where A is the 2×2 matrix from Question 38. Find the matrix of T with respect to the basis $\mathcal{B}$ given in Question 38.

(a) $\begin{bmatrix} 2 & 10 \\ 0 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 10 \\ -3 & 8 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 8 \\ -3 & 10 \end{bmatrix}$ (e) $\begin{bmatrix} 8 & -10 \\ 3 & -3 \end{bmatrix}$

(f) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (h) $\begin{bmatrix} 2 & -3 \\ 0 & -3 \end{bmatrix}$.

40. Find a basis for the real linear space $V = \left\{ \begin{bmatrix} a & a \\ b & b \\ c & c \end{bmatrix} \in \mathbb{R}^{2\times 3} : a, b, c \in \mathbb{R} \right\}$.

(a) $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$ (f) $\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ (g) V has a basis, but it is infinite.

41. Let $A,\ B \in \mathbb{R}^{n\times n}$. We define a map $T : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$ by $T(M) = AMB$ for all $M \in \mathbb{R}^{n\times n}$. Justify whether or not T is a linear transformation. Show all work.

42. Let $T : P_2 \to \mathbb{R}^2$ be given by $T(f) = \begin{bmatrix} 4f(1) \\ f'(4) \end{bmatrix}$, where f' denotes the derivative of the polynomial f; i.e., $T(f) = \begin{bmatrix} 4f(t)\big|_{t=1} \\ \frac{df}{dt}\big|_{t=4} \end{bmatrix}$. You may assume that T is a linear transformation. Using the basis $\{t^2, t, 1\}$ for P_2 and the basis $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^2$, find the matrix of T with respect to these bases.

(a) $\begin{bmatrix} 4 & 4 & 4 \\ 8 & 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 4 & 4 \\ 8 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 8 \\ 4 & 1 \\ 4 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 0 \\ 4 & 1 \\ 4 & 8 \end{bmatrix}$ (e) $\begin{bmatrix} 4 & 4 & 4 \\ 0 & 1 & 8 \end{bmatrix}$ (f) $\begin{bmatrix} 4 & 4 & 4 \\ 8 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

43. Let $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and let $\mathcal{E} = \{e_1, e_2, e_3\}$ be the standard basis of $\mathbb{R}^3$, and assume $\mathcal{B} = \{-e_2,\ e_1,\ e_1 + e_3\}$ is another basis of $\mathbb{R}^3$. Find the matrix which changes basis from $\mathcal{B}$ to $\mathcal{E}$.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (f) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (g) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ (h) $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$.

44. Which of the following sets of vectors form an orthonormal set?

(a) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ -\frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{5}}{3} \\ 0 \\ \frac{2}{3} \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ \frac{\sqrt{5}}{3} \end{bmatrix} \right\}$ (f) none of them.

45. Use the Gram-Schmidt orthogonalization procedure to produce an orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$.

(a) $\left\{ \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ \frac{\sqrt{3}}{\sqrt{5}} \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{2}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \right\}$ (f) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{2}{\sqrt{5}} \\ 0 \\ -\frac{1}{\sqrt{5}} \end{bmatrix} \right\}$ (g) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (h) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{bmatrix} \right\}$.

46. Which of the following matrices is orthogonal?

(a) $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{3} & \frac{1}{2} & -1 \\ -\frac{1}{3} & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (f) $\begin{bmatrix} \frac{2}{3} & \frac{2}{3} & 1 \\ -\frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & -\frac{1}{3} & 0 \end{bmatrix}$ (g) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$.

47. Find the determinant of $\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$.

(a) -15 (b) -1 (c) 1 (d) 0 (e) 2 (f) 3 (g) 5 (h) 7 (i) 9.

48. If A, B and C are $n \times n$ matrices such that $\det(A) = 5$, $\det(B) = 2$, $\det(C) = 4$, find $\det(AB^{-1}C^T)$.

(a) -4 (b) -2 (c) 0 (d) 2 (e) 4 (f) 6 (g) 8 (h) 10 (i) 12.

49. If A is a 5×5 matrix, find $\det(2A)$.

(a) $128\det(A)$ (b) $32\det(A)$ (c) $8\det(A)$ (d) $2\det(A)$ (e) 0
(f) $-2\det(A)$ (g) $-8\det(A)$ (h) $-32\det(A)$ (i) $-128\det(A)$.

50. Let A denote an $n \times n$ matrix. Give the <u>definition</u> of an eigenvalue λ of A.

51. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -3 \\ 0 & 3 & 5 \end{bmatrix}$. The matrix A has an eigenvector $v = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ associated to an eigenvalue λ; find λ. (Hint: the fastest way to do this question does not use any polynomials.)

(a) 0 (b) -1 (c) 1 (d) -2 (e) 2 (f) -3 (g) 3 (h) -4 (i) 4.

52. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ has an eigenvector $v \in \mathbb{R}^3$ associated to an eigenvalue $\lambda = 3$; find $T^4(v)$.

(a) $243v$ (b) $9v$ (c) $81v$ (d) $99v$ (e) $3v$ (f) $27v$ (g) not enough information.

53. Find the characteristic polynomial of $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.

(a) $1 - x$ (b) $x^2 + 4x + 1$ (c) $x^2 + 4x - 1$ (d) $x^2 - 4x + 1$ (e) $x^2 - 4x - 1$
(f) $x^2 - 4x + 5$ (g) $x^2 + 4x - 5$ (h) $x^2 - 4x - 5$ (i) $x^2 + 4x + 5$.

54. Find all the eigenvalues of $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 5 \\ 0 & -6 & 8 \end{bmatrix}$.

(a) 1, 2 & 3 only (b) 2 & 3 only (c) 1 & 3 only (d) 1 & 2 only
(e) 1 only (f) 2 only (g) 3 only (h) A has no eigenvalues in $\mathbb{R}$.

55. The matrix $A = \begin{bmatrix} 0 & 1 & -3 \\ -3 & 4 & -3 \\ 0 & 0 & 3 \end{bmatrix}$ has eigenvalue 3. Find the eigenspace of the eigenvalue 3. Show all work and write clearly.

Remember: most of the test will be based on the homework questions, so finish those questions and look over your solutions to them before the test. Also look over the previous tests and midterms. Reread Page 1 above to see what to bring to the test.