MIDTERM 1 is in PKH 103 in class time, Thurs Oct 6

- See above for date, time and location of MIDTERM 1. It will last 70 minutes.
- The material covered on Midterm 1 is the same as that covered on the homework through the end of §3.1 inclusive PLUS the material from questions 1-3, 5, 34, 48, 49 in §3.2. (Homework is listed at my website: www.uta.edu/math/vancliff/T/F11 .)
- My office hours are: Tues & Thurs 3:30-4:20 pm.
- This test will be, in part, multiple choice, but you do NOT need to bring a scantron form. There will be several choices of answer per multiple-choice question and, for each, only one answer will be the correct one. You should do rough work on the test or on paper provided by me. No calculator is allowed. No notes or cards are allowed.
- When I write a test, I look over the lecture notes and homework which have already been assigned, and use them to model about 85% of the test problems (and most of them are fair game). You should expect 14-20 questions in total.
- A good way to review is to go over the homework problems you have not already done & make sure you understand all the homework well by 48 hours prior to the test. In addition, this information sheet provides some practice problems that are provided to help you study if you have finished all the homework questions. These practice questions do NOT form a model for the test. These questions are intended only to help you identify any gaps in your understanding. In the last 24 hours before the test, reread ALL the homework problems, skim through the lecture notes, & go over these practice questions again.
- Try to keep your eyes on your own work during the test.
- All nontest items will have to be put by the wall or on my desk prior to the start of the test; this includes cell phones, which should be switched off.
- Any student who leaves the room during the test will not be allowed to continue the test. If you finish the test early, then your fellow students would prefer that you NOT distract their concentration by leaving the room early. Instead, stay in your seat, and look over your work and check your answers.
- It is your responsibility to be on time.
- I hope to return the graded test to you on Tues Oct 11 with a solution sheet. We will have a lecture as usual on Tues Oct 11.

The following questions are for your practice for the test to see how I might phrase some questions (especially multiple choice); they will not be graded and do NOT form a model for the test. Finish the homework assignments first, before working them.

PRACTICE QUESTIONS

1. Which of the following systems of equations is useful for determining the function(s) $f(t) = a + bt + ct^2$ whose graph passes through the points $(0,\ 2),\ (1,\ 8),\ (-1,\ 18)$?

(a) $\begin{aligned} a &= -2 \\ a + 4b &= -8 \\ a - b + c &= -18 \end{aligned}$ (b) $\begin{aligned} a &= 2 \\ a + b + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (c) $\begin{aligned} a &= 2 \\ a + 4b &= 8 \\ a - b + c &= 18 \end{aligned}$

(d) $\begin{aligned} a &= -2 \\ a + b + c &= -8 \\ a - b + c &= -18 \end{aligned}$ (e) $\begin{aligned} b &= 2 \\ a + c &= 8 \\ a - b + c &= 18 \end{aligned}$ (f) $\begin{aligned} a + b &= 2 \\ a + c &= 8 \\ b - c &= 18. \end{aligned}$

2. Which of the following systems of equations is useful for determining the function(s) $f(t) = a\cos(2t) + b\sin(2t)$ that satisfy the equation $\dfrac{d^2f}{dt^2} + 2\dfrac{df}{dt} + 3f(t) = 5\cos(2t)$ for all $t \in \mathbb{R}$?

(a) $\begin{aligned} 4a - b &= 0 \\ a + 4b &= -5 \end{aligned}$ (b) $\begin{aligned} 4a + b &= 0 \\ a - 4b &= -5 \end{aligned}$ (c) $\begin{aligned} a + 4b &= 0 \\ a + 4b &= -5 \end{aligned}$ (d) $\begin{aligned} a + 4b &= 0 \\ a + b &= -5 \end{aligned}$

(e) $\begin{aligned} a + b &= 0 \\ a + 2b &= -5 \end{aligned}$ (f) $\begin{aligned} a - b &= 0 \\ a - 2b &= -5. \end{aligned}$

3. Write down the coefficient matrix and the augmented matrix (stating which is which) for the system
$$\begin{aligned} x_1 - x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 &= -1 \\ x_2 - 2x_3 &= 1. \end{aligned}$$

4. Which of the following matrices is not in row-echelon form?

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 4 & 5 & 6 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 8 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 4 & 8 & 8 \\ 0 & 0 & 9 & 10 \\ 0 & 2 & 3 & 4 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

5. If $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, then which of the following is a nontrivial solution of the equation $Ax = 0$?

(a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (g) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (h) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

6. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the set of solutions to $Ax = 0$ geometrically?

   (a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) a line in $\mathbb{R}^3$ (d) a plane in $\mathbb{R}^3$

   (e) a plane in $\mathbb{R}^4$ (f) $\mathbb{R}^3$ (g) $\mathbb{R}^4$ (h) 3 points in $\mathbb{R}^3$.

7. The system of linear equations $\begin{aligned} x_1 + 4x_2 + 3x_3 &= 1 \\ 2x_2 + x_3 &= 1 \\ x_2 + x_3 &= 1 \end{aligned}$ has

   (a) no solution (b) exactly 1 solution (c) infinitely many solutions (d) exactly 2 solutions.

8. If $\begin{bmatrix} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 5 & k \end{bmatrix}$ is the augmented matrix for a system of linear equations, then for which value(s) of $k \in \mathbb{R}$ is the system consistent?

   (a) $k = 1$ (b) $k = 2$ (c) $k = 4$ (d) all $k \in \mathbb{R}$ except $k = 1$

   (e) all $k \in \mathbb{R}$ except $k = 2$ (f) all $k \in \mathbb{R}$ except $k = 4$.

9. Justify or disprove: if a given vector b is a linear combination of the columns of a matrix A, then the equation $Ax = b$ is consistent.

10. If $\begin{bmatrix} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$ is the augmented matrix for a system of linear equations, then the solution set of the system in vector form is:

    (a) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (b) $x = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (c) $x = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

    (d) there are no solutions (e) not enough information given to answer question.

11. Which of the following statements is always true?

    (a) A homogeneous system of linear equations is always consistent.
    (b) A homogeneous system of linear equations is never consistent.
    (c) A homogeneous system of linear equations always has nontrivial solutions.
    (d) If A is a 3×2 matrix, then A must have a column without a pivot.

12. Compute $\begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

    (a) $\begin{bmatrix} 2 & 2 & 7 \\ -2 & 4 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 6 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & -2 \\ 2 & 4 \\ 7 & 10 \end{bmatrix}$

    (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} -1 & 2 & -3 \\ -1 & 6 & -5 \end{bmatrix}$ (f) $\begin{bmatrix} -1 & -1 \\ 2 & 6 \\ -3 & -5 \end{bmatrix}$ (g) cannot be multiplied.

13. Noting that the vector $v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is a particular solution to the system $\begin{aligned} 2x_1 - x_2 + x_3 &= 0 \\ 3x_1 + 2x_2 - x_3 + x_4 &= 1 \\ 5x_1 + 8x_2 - 5x_3 + 3x_4 &= 3, \end{aligned}$

find *all* solutions to the system.

(a) $v + s\begin{bmatrix} 1 \\ 0 \\ -2 \\ -5 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ (b) $v + s\begin{bmatrix} 1 \\ 0 \\ 2 \\ -5 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ (c) $v + s\begin{bmatrix} 1 \\ 0 \\ -2 \\ -5 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(d) none of the above.

14. Given that the reduced row-echelon form of $\begin{bmatrix} 1 & 2 & 12 \\ 0 & 1 & 5 \\ 1 & 1 & k \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & k-7 \end{bmatrix}$,

for which value(s) of k is $\begin{bmatrix} 12 \\ 5 \\ k \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$?

(a) all $k \in \mathbb{R}$ except 7 (b) $k = 0$ only (c) $k = 7$ only (d) $k = 0$ & $k = 7$ only

(e) all $k \in \mathbb{R}$ except 0 & 7 (f) all $k \in \mathbb{R}$ (g) there is no such k (h) not enough information given.

15. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{cases} \begin{bmatrix} x_1 \\ \frac{x_2^2}{x_1} \end{bmatrix} & \text{if } x_1 \neq 0 \\ \begin{bmatrix} 0 \\ x_2 \end{bmatrix} & \text{if } x_1 = 0. \end{cases}$

Show that there exist $u, v \in \mathbb{R}^2$ such that $T(u+v) \neq T(u) + T(v)$. (Hint: consider $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.)

16. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right)$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \left(\begin{bmatrix} -1 \\ 6 \end{bmatrix}\right)$, find $T\left(\begin{bmatrix} 5 \\ -3 \end{bmatrix}\right)$.

(a) $\begin{bmatrix} 13 \\ 7 \end{bmatrix}$ (b) $\begin{bmatrix} -7 \\ 43 \end{bmatrix}$ (c) $\begin{bmatrix} -13 \\ -7 \end{bmatrix}$ (d) $\begin{bmatrix} 10 \\ 15 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (f) $\begin{bmatrix} 2 \\ 23 \end{bmatrix}$.

17. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that $T(u) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(v) = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, find $T(2u - 3v)$.

(a) $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ (f) not enough information is given.

18. Find the matrix of the function $T : \mathbb{R}^2 \to \mathbb{R}$, where $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_2^2$.

(a) $\begin{bmatrix} x_1 & x_2^2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & x_2^2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & x_2 \end{bmatrix}$ (e) $\begin{bmatrix} x_2^2 \end{bmatrix}$ (f) no such matrix.

19. Suppose a line ℓ in $\mathbb{R}^2$ contains the vector $u = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$ of length 1. Find the matrix of the (orthogonal) projection map onto ℓ.

(a) $\begin{bmatrix} 0.64 & -0.48 \\ -0.48 & 0.36 \end{bmatrix}$ (b) $\begin{bmatrix} 0.48 & -0.64 \\ -0.64 & 0.36 \end{bmatrix}$ (c) $\begin{bmatrix} 0.36 & -0.48 \\ -0.48 & 0.64 \end{bmatrix}$

(d) $\begin{bmatrix} 0.48 & -0.36 \\ -0.36 & 0.64 \end{bmatrix}$ (e) $\begin{bmatrix} 0.64 & -0.36 \\ -0.36 & 0.48 \end{bmatrix}$ (f) no such matrix.

20. Find the shear matrix that transforms $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 16 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 \\ 0 & 5 \end{bmatrix}$ (f) no such shear.

21. If $A = \begin{bmatrix} k & 2 \\ -3 & 4 \end{bmatrix}$, then for which value(s) of k is A not invertible?

(a) 2 (b) $\frac{3}{2}$ (c) -2 (d) $-\frac{3}{2}$ (e) 0 (f) 0 & 2.

22. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 2 & 1 & 4 \end{bmatrix}$, then which of the following is the second row of A^{-1}?

(a) 1 1 1 (b) 0 0 1 (c) 4 −2 −3 (d) 1 2 −3 (e) 2 3 4 (f) 4 5 −6
(g) 0 −2 1 (h) 0 2 1 (i) A^{-1} does not exist.

23. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 7 \\ 10 \\ 9 \end{bmatrix}$. Describe $\text{span}(v_1, v_2, v_3) = \mathbb{R}v_1 + \mathbb{R}v_2 + \mathbb{R}v_3$.

(a) a point (b) a line (c) a plane (d) $\mathbb{R}^3$ (e) exactly 3 points
(f) exactly 3 vectors emerging from the origin in $\mathbb{R}^3$.

24. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the kernel of A geometrically?

(a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) 3 points in $\mathbb{R}^3$
(d) a line in $\mathbb{R}^3$ (e) a plane in $\mathbb{R}^3$ (f) a plane in $\mathbb{R}^9$ (g) $\mathbb{R}^3$ (h) $\mathbb{R}^4$.

25. If $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$, then which of the following best describes the image of A geometrically?

(a) The zero vector in $\mathbb{R}^2$ (b) The zero vector in $\mathbb{R}^3$ (c) 3 points in $\mathbb{R}^3$

(d) a line in $\mathbb{R}^3$ (e) a plane in $\mathbb{R}^3$ (f) a plane in $\mathbb{R}^9$ (g) $\mathbb{R}^3$ (h) $\mathbb{R}^4$.

26. The set $U = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ 5 \end{bmatrix} : x_1, x_2 \in \mathbb{R} \right\}$ is not a subspace of $\mathbb{R}^3$ because

(a) U is not a subset of $\mathbb{R}^3$ (b) U is empty (c) U contains $\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$ (d) U contains the origin

(e) U does not contain the origin (f) trick question: U is a subspace of $\mathbb{R}^3$.

27. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, $v_3 = \begin{bmatrix} 9 \\ 2 \\ 33 \end{bmatrix}$. Given that $\begin{bmatrix} 5 \\ -7 \\ 1 \end{bmatrix}$ is in the kernel of the matrix $A = \begin{bmatrix} 1 & 2 & 9 \\ 1 & 1 & 2 \\ -1 & 4 & 33 \end{bmatrix}$, find a nontrivial relation among v_1, v_2 and v_3.

Show some work; write clearly.

28. Define (i) subspace of $\mathbb{R}^n$ (ii) span of m vectors in $\mathbb{R}^n$ (iii) kernel of a linear transformation (iv) image of a linear transformation.

Remember: most of the test will be based on the homework questions, so finish those questions and look over your solutions to them before the test. Reread Page 1 above to see what to bring to the test.